The four subspaces as orthogonal complements

Calculating basis for an orthogonal complement - How? Calculating orthonormal basis for an orthogonal complement - How?

$A\bar{x} = \bar{b}$ has a solution, $\bar{b}$ is in the column space of $A,$ if and only if $H\bar{b} = \bar{0}$, $\bar{b}$ is in the nullspace of $H.$ What's in H ? Why?

If $A\bar{x} = \bar{b}$ has a solution for a given $\bar{b}$, then there is always a (unique) solution in the row space of $A.$ How does that work?

Dimensions of the 4 subspaces.

Given an $m \times n$ matrix $A,$ and a vector $\bar{x}$ in $R^n,$ we can write $\bar{x} = \bar{x}_{\text{row}} + \bar{x}_{null}$, where the vectors on the right are in the row and null spaces of A respectively. How is $\bar{x}_{\text{row}}$ related to $\bar{x}$ via projection? How could you find matrices P, Q such that $\bar{x}_{\text{row}} = P\bar{x}$ and $\bar{x}_{null} = Q\bar{x}.$

The null space of A is the same as the null space of $A^TA.$ How does that work?

Orthogonal projections:

Onto a one-dimensional subspace

Onto a subspace.

How to calculate a projection, how to calculate the projection matrix, how to identify a projection (what is the definition?).

Projection when you have an orthonormal basis $Q.$ $\bar{p} = QQ^T\bar{x}$ why?

What is the significance of writing $\bar{b} = \bar{p} + (\bar{b} - \bar{p})$? We have decomposed $\bar{b}$ into a sum of two vectors that below to what subspaces?

Least-squares solution of a linear system $A\bar{x} = \bar{b}.$ The "solution" $\bar{x}$ has the property that $A\bar{x} = what?$

The normal equations - what are they, where do they come from, what do they express?

Matrix Q with orthonormal columns - $Q^TQ = I$, preserves dot products

Gram-Schmidt, leads to $A = QR$ decomposition. Since $\text{col}(A) = \text{col}(Q)$ by construction, the normal equations for calculating the least squares solution of $A\bar{x} = \bar{b}$ can be alternatively written $Q^T(\bar{b} - A\bar{x}) = \bar{0}$, $Q^T\bar{b} = Q^TQR\bar{x}$ so $Q^T\bar{b} = R\bar{x}.$ Of course $\bar{p}$ itself is simply $\bar{p} = QQ^T\bar{b}.$

Determinants:

Basic properties, elimination steps and the value of the determinant.

Evaluating a determinant using successive elimination and expansion.

What the determinant determines

$\det(AB) = \det A \det B$, $\det A^T = \det A$

Total expansion of a determinant: the big formula

$A = \det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ In the expansion of $\det A = \ldots + (\quad)bfg + \ldots$ does $+$ or $-$ go in the

parentheses?
Expansion (across rows or down columns) by cofactors
The cofactor matrix C and its property $AC^T = (\det A)I$
Inverses using the cofactor matrix. Cramer's rule.