Math 441 Exam 1

1. a) Given a vector $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ find A so that $A\bar{x} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ \vdots \\ x_n - x_{n-1} \end{bmatrix}$. (Describe the form and state the dimensions of the matrix A)

b) Express the vector $\begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ \vdots \\ x_n - x_{n-1} \end{bmatrix}$ as a linear combination

$$\begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ \vdots \\ x_n - x_{n-1} \end{bmatrix} = x_1 \begin{bmatrix} \bar{u}_1 \\ \\ \\ \end{bmatrix} + x_2 \begin{bmatrix} \bar{u}_2 \\ \\ \\ \end{bmatrix} + \ldots + x_n \begin{bmatrix} \bar{u}_n \\ \\ \\ \end{bmatrix}$$ (find the vectors $\bar{u}_1, \bar{u}_2, .., \bar{u}_n$)

c) What is the connection between part a) and part b)?

2. Using Guassian elimination along with back substitution on the augmented matrix, solve the linear system

$$A\bar{x} = \begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}.$$

a) Without doing any further calculations, write the LDU decomposition of the matrix A.

b) What sequence of elimination matrices $E_1, E_2, .., E_k$ multiplying A on the left in turn, takes us from A to U in part a) (so that $E_k E_{k-1} \cdot \cdot E_1 A = U$)

3. If A is a 4×4 matirx,

a) What matrix $E,$ upon the multiplication $EA,$ has the effect on A of subtracting 3 times row 2 of A from row 4 of A?

b) What is the effect on A if we perform the operation AE using the same matrix E from part a)

c) Without doing any calculations, what is E^{-1} ?

4. Using Gauss-Jordan elimination, in the space below find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

and check your answer. Then, considering only those calculations, answer the questions that follow:

i) When we go from A via Gaussian elimination to the upper triangular matrix U, what single matrix M, multiplying on the left, would turn A into U? Why? What is the relationship of this M to L from the LU decomposition.

ii) Changing a_{23} to what value would cause A^{-1} not to exist?

5. If we know that $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, solve $A\bar{x} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

(Hint: first multiply both sides of $A\bar{x} = \bar{b}$ on the left by P, do not calculate A !)