Research Prospectus

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1 Problems Motivating The Research

The research project is motivated by hamiltonian line graph problems and supereulerian problems. All graphs considered are undirected, finite and loopless, but parallel edges are allowed. Throughout this prospectus, we follow [1] for undefined terms and notations.

1.1 Motivation of Research on hamiltonian line graphs

A graph is hamiltonian if it has a spanning cycle. A line graph $L(G)$ of a graph G is a graph obtained by taking $E(G)$ to be the vertex set $V[L(G)]$, and any two vertex of $V[L(G)]$ are adjacent if and only if they are adjacent in G as edges. Closed Spanning trail problems are really fundamental and important in graph theory and the two main types of closed spanning trails studied by people are edge-disjoint closed spanning trail(eulerian subgraph) and vertex-disjoint closed spanning trail(hamiltonian cycle).

Harary and Nash-Williams [2] pointed out the relation between eulerian subgraph of a graph G and the hamiltonicity of its line graph.

Theorem 1.1 (Harary and Nash-Williams, [2]) *Let G be a connected graph with at least 3 edges. Then $L(G)$ is hamiltonian if and only if G has an Eulerian* *subgraph H such that $E(G-V(H))=\emptyset$.*

In the study on hamiltonian line graph problems, an interesting question is whether high connectivity certainly leads to hamiltonicity of line graphs. This is not true for general graphs if we consider the complete bipartite graph $K_{m+1,m}$. $K_{m+1,m}$ is m connected, but yet does not contain a hamiltonian cycle. However, researchers still proposed the following well-known and fascinating conjectures.

Conjecture 1 *(i) (Thomassen [27]) Every 4-connected line graph is hamiltonian.*
(ii) (Matthews and Sumner [28]) Every 4-connected claw-free graph is hamiltonian.
(iii) (Kučzel and Xiong [29]) Every 4-connected line graph is hamiltonian-connected.
(iv) (Ryjáček and Vrána [30]) Every 4-connected claw-free graph is hamiltonian-connected.

Among the efforts made on studying hamiltonian line graphs with connectivity restrictions, people made some progress as listed below.

Theorem 1.2 *Let G be a graph.*
(i) (Zhan, Theorem 3 in [31]) If $\kappa(L(G))\geq 7$, then $L(G)$ is hamiltonian-connected.
(ii) (Kaiser and Vrána [32]) If $\kappa(L(G))\geq 5$ and $\delta(L(G))\geq 6$, then $L(G)$ is hamiltonian.

These results are encouraging and motivated us to further study this topic. A natural generalization of hamiltonicity is the s-hamiltonicity of a graph which is defined as

Definition 1 *A graph G of order $n\geq 3$ is called* s-**hamiltonian** $(0\leq s\leq n-3)$, *if for any $X\subseteq V(G)$ with $|X|\leq s$, $G-X$ is hamiltonian.*

By definitions, if $s\geq 1$, then s-hamiltonian graphs are $(s-1)$-hamiltonian, and s-hamiltonian graphs are hamiltonian. It is well known that if a graph G is s-hamiltonian, then G is $(s+2)$-connected. But the converse is not generally true, $K_{m,m+1}$ is again a counterexample. Hence, it is natural to ask the question:

How big does the connectivity need to be to guarantee the s-hamiltonicity of line graphs? Is it still true that $L(G)$ is s-hamiltonian if and only if $L(G)$ is $(s+2)$-connected for all integers $s \geq k$? Broersma and Veldman in [39] considered this question within triangular graphs. They define,

Definition 2 *For an integer $k \geq 0$, a graph G is k-**triangular** if every edge of G lies in at least k triangles of G.*

And they obtained the following.

Theorem 1.3 *(Broersma and Veldman, [39]) Let $k \geq s \geq 0$ be integers and let G be a k-triangular simple graph. Then $L(G)$ is s-hamiltonian if and only $L(G)$ is $(s+2)$-connected.*

Broersma and Veldman in [39] proposed an open problem of determining the range of integral values s such that within triangular graphs, $L(G)$ is s-hamiltonian if and only $L(G)$ is $(s+2)$-connected. This problem was first settled by Chen et al in [40]. Later, it is further extended in [41].

Theorem 1.4 *Each of the following holds.*
(i) (Chen et al., Theorem 1.2 in [40]) Let k and s be positive integers such that $0 \leq s \leq \max\{2k, 6k-16\}$, and let G be a k-triangular simple graph. Then $L(G)$ is s-hamiltonian if and only $L(G)$ is $(s+2)$-connected.
(ii) (Theorem 1.3 in [41]) Let G be a connected graph and let $s \geq 5$ be an integer. Then $L(G)$ is s-hamiltonian if and only if $L(G)$ is $(s+2)$-connected.

In the view of the series of conjectures in Conjecture 1 and, results presented above, we propose the following.

Conjecture 2 *Let G be a connected graph and let s be an integer.*
(i) Let $s \geq 2$. Then $L(G)$ is s-hamiltonian if and only if $\kappa(L(G)) \geq s+2$.
(ii) Let $s \geq 1$. Then $L(G)$ is s-hamiltonian-connected if and only if $\kappa(L(G)) \geq s+3$.
(iii) If G is $K_{1,3}$-free, then for $s \geq 2$ G is s-hamiltonian if and only if G is $(s+2)$-connected.

(iv) If G is $K_{1,3}$-free, then for $s \geq 1$ G is s-hamiltonian-connected if and only if G is $(s+3)$-connected.

In some sense, taking line graph of a given graph is similar to taking derivatives of functions, it provides a reasonable way to study the original graph. Hence, just like taking the n-th derivative of a function, consider the change of certain properties as we take the n-th line graph of a graph is a natural step forward. We denote the n-th line graph of G by $L^n(G)$, that is, $L^n(G) = L^{n-1}(G)$. For a property P and a connected nonempty graph G, the ***P*-index** of G, denoted $P(G)$, is defined by

Definition 3

$$P(G) = \begin{cases} \min\{k : L^k(G) \textit{ has property } P\} & \textit{if at least one such integer } k \textit{ exists} \\ \infty & \textit{otherwise} \end{cases}$$

If the property P is hamiltonian, then it is the hamiltonian index first introduced by Chartrand [3] which is usually denoted by $h(G)$. He also proved in [5] that the index gives a stable indication of when the hamiltonicity will emerge, which means that the hamiltonicity will not go away once it appears as we take line graphs.

Later, Clark and Wormald [4] introduced more hamiltonian like indices of graphs, edge-hamiltonian index, pancyclic index, vertex-pancyclic index, edge-pancyclic index, hamiltonian-connected index. Lai and Shao [7] proved that these indices are also stable under the operation of taking line graph. Clark and Wormald [4] proved the existence of these indices as finite numbers. However, generally speaking to determine these indices of a graph is hard. For example Ryjáček, Woeginger and Xiong [6] showed that determining the hamiltonian index of a graph is NP-complete.

Since these indices are proved to be attainable, stable and non-trivial, they are interesting graph invariants to be studied. What graphical structure is determining these indices and what is the relation between these indices and usual graph invariants? Our curiosity on these questions motivated our research on hamiltonian-like indices.

For a graph G that $n = |G| \geq 3$, G is said to be panconnected if for any pair of indices $u, v \in V(G)$ and for any integer $d(u, v) \leq s \leq n - 1$, there is always an (u, v)-path with length s. Panconnectivity is first considered by A.Yousef and W.James [21]. As we can see by definition that if G is panconnected then G must be hamiltonian connected, panconnectedness is even stronger than the hamiltonian connectedness. And the panconnectd index $pc(G)$ is defined as the smallest number k such that $L^k(G)$ is panconnected.

A bridge divalent path is a path in a graph such that the internal vertices has degree 2. And $\ell(G) = \max\{m : G$ has a divalent path of length m that is not both of length 2 and in a $K_3\}$. In studies on hamilton index, edge-hamiltonian index, pancyclic index, vertex-pancyclic index, edge-pancyclic index and hamiltonian-connected index(see [5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] for examples), researches found $\ell(G)$ usually determines the upper bounds of these indices. It has been observed that $\ell(G)$ does not only give the upper bound of hamiltonian index, hamiltonian-connected index, s-hamiltonian index and pancyclic, but the bounds are also sharp[10, 12, 13, 16, 17, 20]. This inspires us to consider whether $\ell(G)$ also plays an important role in determining $pc(G)$.

1.2 Motivation of Research on supereulerian properties of graphs

As we know that graph theory can be originated to the time when Euler proposed the *Seven bridges problem*, finding eulerian subgraph is always an elementary and profound topic in the study of graph theory.

In [45], Boesch et al. first raised the problem of characterizing supereulerian graphs. They remarked that such a problem would be difficult. In [47], Pulleyblank confirmed the remark by showing that the problem to determine if a graph is supereulerian, even within planar graphs, is NP-complete. Jaeger [48] first proved that every 4-edge-connected graph is supereulerian (for more details, see [49]). In [22], Catlin introduced collapsible graphs as a tool to study supereulerian graphs. Since Catlin proved that ([22]) G is supereulerian if and only if G/H

is supereulerian, where H is a collapsible subgraph of G, this gives us an useful tool to do induction. And more important, we are often able to reduce the graphs we need to consider to finite number of them. Using the reduction method of collapsible graphs, lots of new results emerge on supereulerian problems([50], [51]). A graph G is hamiltonian-connected if for any $u, v \in V(G)$, there is a hamiltonian (u, v)-path. As a similar but more general idea, Luo *et al* [23] defined a graph G to be eulerian-connected if for any $u, v \in V(G)$, there is an eulerian (u, v)-path. By definition, a graph is eulerian-connected if it is hamiltonian-connected.

To further generalize this idea, Lai *et al* [24] defined the supereulerian width $\mu'(G)$ of a graph G. For a graph G and an integer $s > 0$ and for $u, v \in V(G)$ with $u \neq v$, an $(s; u, v)$-trail-system of G is a subgraph H consisting of s edge-disjoint (u, v)-trails. $\mu'(G)$ of a graph G is the largest integer s such that for any $u, v \in G$, G has a spanning $(k; u, v)$-trail-system, for any integer k with $1 \leq k \leq s$. By this definition, a graph G is eulerian connected if $\mu'(G) \geq 2$. Since $\mu'(G)$ is such a natural generalization of supereulerian problems, this topic aroused our curiosity.

To settle an open problem of Bauer ([25, 26]), Catlin proved Theorem [22](i) below, which was recently extended by Li *et al.* in [24].

Theorem 1.5 *Let G be a simple graph on n vertices.*
(i) (Catlin, Theorem 9(ii) of [22]) If $n \geq 17$ and $\delta(G) \geq \frac{n}{4} - 1$, then $\mu'(G) \geq 2$.
(ii) (Li et al, Theorem 5.3(i) of [24]) For any positive integers p and s with $p \geq 2$, there exists an integer $N = N(s, p)$ and a finite family F_0 of graphs with supereulerian width at most s such that if $\delta(G) \geq \frac{n}{p} - 1$, then either $\mu'(G) \geq s+1$, or G is contractible to a member in F_0.

These results inspired us to study $\mu(G)$ by using the degree condition. If we think about it this way, let $a_0 = \frac{1}{p}$ and $b_0 = -1$, then the degree condition $\delta(G) \geq \frac{n}{p} - 1$ in (ii) is actually $\delta(G) \geq a_0 n + b_0$. Hence, we further generalize the degree condition to consider general $an + b$ where $o \leq a \leq 1$.

Lai *et al* [24] extended idea of collapsible graphs to s-collapsible graphs which is defined as

Definition 4 *A graph G is* **s-collapsible** *if for any subset $R \subseteq V(G)$ with $|R| \equiv 0$ (mod 2), G has a spanning subgraph Γ_R such that*
(i) both $O(\Gamma_R) = R$ and $\kappa'(\Gamma_R) \geq s-1$, and
(ii) $G - E(\Gamma_R)$ is connected.

A graph is **C_s-reduced** if it contains no nontrivial subgraph in C_s. It is shown in [24] that every graph G has a unique collection of maximally s-collapsible subgraphs $H_1, H_2, \cdots, H_c$, and the graph $G'_s = G/(\cup_{i=1}^{c} E(H_i))$ is C_s-reduced, which is called the **C_s-reduction** of G. Using the C_s-reduction technique, we are going to attack the supereulerian width problem of graphs.

2 Progress

2.1 Panconnected index of a graph

Let bridge divalent $(3,t)$-path be the bridge divalent path with one end has degree 3 and the other has degree t. We obtained

Theorem 2.1 *Let G be a connected graph not isomorphic to a path, a cycle and $K_{1,3}$. Then $pc(G) \leq \ell(G) + 2$. Furthermore, if $\ell(G) \geq 2$, then $pc(G) = \ell(G) + 2$ if and only if for some integer $t \geq 3$, G has a bridge divalent $(3,t)$-path of length $\ell(G)$.*

The bound is sharp in the sense of a family of extremal graphs with the structure of having a bridge divalent $(3,t)$-path of length $\ell(G)$ is given. An edge cut X of G is an **essential edge cut** if at least two components of $G - X$ has at least one edge respectively. A graph G is **essentially k-edge-connected** if G does not have an essential edge cut with less than k edges. With an additional restriction of being triangular, we obtained

Theorem 2.2 *Let G be a connected graph not isomorphic to a path, a cycle and $K_{1,3}$. If every edge of G lies in a cycle of length at most 3, in G, then $L(G)$ is panconnected if and only if G is essentially 3-edge-connected.*

2.2 Supereulerian width of a graph

Relaxing the degree restriction on a sigle vertex to any two nonadjacent vertices and by using the C_s-reduction, we obtained

Theorem 2.3 *For any real numbers a, b with $0 < a < 1$ and any integer $s > 0$, there exists a finite family $F = (a, b, s)$ such that for any simple graph G with $n = |V(G)|$, if for any pair of nonadjacent vertices u and v, $\max\{d_G(u), d_G(v)\} \geq an + b$, then $\mu'(G) \geq s + 1$ if and only if G is not contractible to a member in F.*

And we also considered the following interesting particular case when $a = \frac{1}{4}$ and $b = -\frac{3}{2}$.

Theorem 2.4 *For a simple graph G with $|V(G)| = n \geq 141$ and $\kappa'(G) \geq 3$, if for any pair of nonadjacent vertices u and v, $\max\{d_G(u),\ d_G(v)\} \geq \frac{n}{4} - \frac{3}{2}$, then $\mu'(G) \geq 3$ if and only if G is not contractible to $K_{3,3}$.*

The reason we consider this case is because $a = \frac{1}{4}$ and $b = -\frac{3}{2}$ gave us a first nontrivial graph $K_{3,3}$ in F. By adjusting a and b, one can continue to discover more graphs in the finite family F.

3 Future Work

3.1 Forbidden subgraphs in hamiltonian line graphs

Another direction of considering hamiltonian line graphs is to consider the restriction of certain forbidden subgraphs. Let Z_k denote the graph derived from identifying one end vertex of a path with k vertices with one vertex of a triangle. The following are some results have been found considering the forbidden graph Z_k.

Theorem 3.1 *Let Q^* be the graph obtained from the Petersen graph by adding one pendant edge to each vertex. Let G be a 3-connected simple claw-free graph.*
(i) (Brousek, Ryjáĉêk and Favaron, [33]) If G is Z_4-free, then G is hamiltonian.

(ii) (Lai [34]) If G is Z_8-free, then G is hamiltonian. Moreover, the graph Q^* indicates the sharpness of this result.
(iii) (J. Fujisawa, [35], see also Ma et al [36]) If G is Z_9-free graph. Then G is hamiltonian unless G is the line graph of Q^*.

There is another type of forbidden subgraph which also drew our attention, the hourglass. An hourglass is the graph consisting of two triangles meeting in exactly one vertex. Results on this restriction are

Theorem 3.2 *(Broersma et al., Theorem 6 in [37]) Every 4-connected hourglass-free claw-free graph is hamiltonian.*

Theorem 3.3 *(Kriesell, Corollary 4 in [38]) Every 4-connected hourglass-free line graph is hamiltonian-connected.*

Before we work on the general cases in Conjecture 2, we will try to find out whether the following is true for Z_8-free graphs.

Conjecture 3 *Let $L(G)$ be a Z_8-free line graph and $s \geq 0$ be an integer. Then each of the following holds.*
(i) For any integer $s \geq 1$, $L(G)$ is s-hamiltonian if and only if $\kappa(L(G)) \geq s+2$.
(ii) For any integer $s \geq 0$, $L(G)$ is s-hamiltonian-connected if and only if $\kappa(L(G)) \geq s+3$.

And on hourglass-free graphs we will try the following.

Conjecture 4 *Let $L(G)$ be an hourglass-free line graph and s be an integer. Each of the following holds.*
(i) If $s \geq 2$, then $L(G)$ is s-hamiltonian if and only if $\kappa(L(G)) \geq s+2$.
(ii) If $s \geq 1$, then $L(G)$ is s-hamiltonian-connected if and only if $\kappa(L(G)) \geq s+3$.

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