Two binomial coefficient analogues

by

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We will discuss two interesting analogues of the binomial coefficients. The first is called a q-analogue because it depends on a parameter q. The standard q-analogue of a nonnegative integer n is the polynomial

$$[n] = 1 + q + q^2 + \cdots + q^{n-1}.$$

Note that plugging $q = 1$ into $[n]$ one gets back n itself. Similarly define a q-factorial by

$$[n]! = [1][2]\cdots[n].$$

Finally, the q-binomial coefficients are defined by

$$\left[\begin{array}{c} n \\ k \end{array}\right] = \frac{[n]!}{[k]![n-k]!}$$

where $0 \le k \le n$. It turns out that these are polynomials in q with many beautiful combinatorial properties including connections with lattice paths, integer partitions, and vector spaces over finite fields. The first half of this talk will survey some of these results.

Our second analogue will in involve the famous Fibonacci numbers, defined by $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \ge 2$. Consider a corresponding fibotorial

$$F_n^! = F_1 F_2 \cdots F_n$$

and the fibonomial coefficients

$$\binom{n}{k}_F = \frac{F_n^!}{F_k^! F_{n-k}^!}.$$

Since these fractions are always integers, one could ask what they count. Using tilings and integer partitions, Sagan and Savage have given the first simple combinatorial interpretation for $\binom{n}{k}_F$ and this will be the topic of the second half of the talk. It will end with an open question about a Fibonacci analogue for the Catalan numbers.