Directions: Show all work. No credit for answers without work. Unless specifically asked for a numerical answer, you may leave your answers in terms of factorials, permutation numbers, and binomial coefficients.

1. [4 points] Determine the number of non-negative integral solutions to the following.

   (a) \( x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 80, \) with \( x_3 \geq 8. \)

   \[ \begin{align*}
   \text{# solns to} & \quad x_1 + \ldots + x_6 = 72; \quad x_1, x_2, \ldots, x_6 \geq 0 \\
   \Rightarrow & \quad 72 \text{ stars, 5 bars} \Rightarrow \quad \binom{77}{5} = 19,757,815
   \end{align*} \]

   (b) \( x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 80, \) with \( x_3 \geq 8 \) and \( x_5 \leq 50. \)

   Take (a) and subtract #solns with \( x_3 \geq 8 \) and \( x_5 \geq 51. \)

   \[ \begin{align*}
   \text{# solns to} & \quad x_1 + \ldots + x_6 = 21; \quad x_1, \ldots, x_6 \geq 0 \\
   \Rightarrow & \quad 21 \text{ stars, 5 bars} \Rightarrow \quad \binom{26}{5} = 19,692,035
   \end{align*} \]

   Answer: \( \binom{77}{5} - \binom{26}{5} = 19,757,815. \)

2. [3 points] A company wishes to order \( s \) sandwiches for their annual party from a menu that lists \( k \) types of sandwich. How many ways are there for the company to complete its order?

   \[ \begin{align*}
   & \quad \text{# stars: } s \quad \text{# bars: } k - 1 \\
   & \quad \binom{s + k - 1}{k - 1} \quad \text{or} \quad \binom{s + k - 1}{s}
   \end{align*} \]

3. [3 points] Find \( \mathcal{P}\{1, 2, 3\}. \)

   \[ \mathcal{P}\{1, 2, 3\} = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\} \} \]

   Note: this question asks for the set \( \mathcal{P}\{1, 2, 3\}, \) not its size \( |\mathcal{P}\{1, 2, 3\}|. \)