Section R2 E Special Classes of Functions

1. Linear Functions – graph is a line

Traditionally we see the notation \( y = mx + b \), but to write this as a function, we say \( y = f(x) \) and \( f(x) = mx + b \) where \( m \) = slope and \( b \) = \( y \)-intercept \((0, b)\).

If we think about slope, \( m \), in particular and recall that \( m = \frac{\Delta x}{\Delta y} \), we can change its notation to something more function-looking:

Point 1: \( \left( x_1, y_1 \right) = \left( x_1, f(x_1) \right) \)

Point 2: \( \left( x_2, y_2 \right) = \left( x_2, f(x_2) \right) \)

Therefore, \( m = \frac{\Delta x}{\Delta y} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_1) - f(x_2)}{x_2 - x_1} \)

Interpreting data from a linear graph is done by relating an independent variable, say \( x \), to the dependent function (or measured quantity) \( f(x) \).

Your textbook (pg. 61 – 63)j shows specifically how \( x = \) height (inches) is related to \( f(x) = \) weight = 4.5\( x \) – 140 (in pounds).

This gives graph coordinates:
\( (x, f(x)) = (\text{height}, \text{weight}) \)

Likewise, we see where
\( x = \) production cost of manufacturing a unit of something
is related to
\( C(x) = \) total cost = production + overhead = 90\( x \) + 2000
2. Piecewise Linear Functions – graph is a grouping of parts of lines that cannot be written as a single expression

These are usually expressed with tiny intervals of domain associated with each part.

Ex: \( f(x) = \begin{cases} x, & 0 \leq x < 4 \\ 5, & 4 \leq x < 6 \\ x, & 0 \leq x < \infty \end{cases} \)

Note how we literally graph in pieces, hence the name “piecewise.”

There are 2 exceedingly important piece-wise functions that we use a great deal in calculus.

1. Absolute Value function: \( f(x) = \|x\| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases} \)
2. The Heaviside function: \( f(x) = \begin{cases} \ 0, & x < 0 \\ 1, & x \geq 0 \end{cases} \)
3. Quadratic Functions – these are what we typically call the parabolas and they have a form that looks like \( f(x) = ax^2 + bx + c \). But it may be more useful to write them as \( f(x) = a(x-h)^2 + k \).

For Example: \( a(x^2 - 2hx + h^2) + k = ax^2 - 2ahx + ah^2 + k \)

or \( f(x) = ax^2 - 2ahx + ah^2 + k \)

\( f(x) = ax^2 + bx + c \) then indicates: \( a = a, \ b = -2ah, \ c = ah^2 + k \)

The parent parabolic form is \( f(x) = x^2 \)
Recall: \( f(x) = a(x-h)^2 + k \)

Here, the vertex is \((h, k)\) and the sign of \(a\) indicates which way the parabola opens. Positive \(a\) means the parabola opens up and negative \(a\) means the parabola opens down.

The value of \(a\) also alters the nature of our known points: 
\[
(1, 1) \Rightarrow (1, a) \\
(-1, 1) \Rightarrow (-1, a) \quad \text{and so on.} \\
(2, 4) \Rightarrow (2, 4a)
\]

4. **Square Root Functions** – \( y = \sqrt{x} \) in a broad sense, this function captures the positive branch of \( y^2 = x \) (the sideways parabola).
And if you look at the point forms you just see where the x and y's seem to change places

<table>
<thead>
<tr>
<th>( y = x^2 )</th>
<th>( y = \sqrt{x} )</th>
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<tbody>
<tr>
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<td>(4, 16)</td>
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NOTE: positive arm ONLY

There are many other kinds of functions which we will discuss in detail, particularly in sections 1.1 and 1.2 of your calculus book!