SAMPLE FINAL TEST

This is an excerpt from the previous Sample Tests. The actual Final Test will be considerably shorter.

Test #1 material

Ex. 1. Each of the following differential equations is of one of the following form: linear, separable, homogenous, Bernoulli, or exact. Solve each of these using appropriate method.

(a) \( y' = \frac{e^{-x} + e^x}{3 + 4y}, \) \( y(0) = 1 \)

(b) \( \frac{u}{x} + 6x + (\ln x - 2) \frac{du}{dx} = 0, \) \( x > 0 \)

(c) \( ty' - y = t^2 e^{-t}, \) \( t > 0 \)

(d) \( \frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy} \)

(f) \( \frac{dy}{dx} + y = \frac{1}{1 + e^x} \)

Ex. 2. Without solving, determine the largest interval in which the initial value problem \((x^2 - x - 6)y' + y \cos x = e^x, \) \( y(2) = 0, \) has a unique solution.

Ex. 3. A tank with a capacity of 500 gal originally contains 200 gal of water with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at the rate of 2 gal/min. Write down an initial value problem (ODE plus initial condition) giving the amount of salt in the tank at any time during the first hour. Do not solve the equation. Remember to give the initial condition.

Test #2 material

Ex. 4. Find the general solution for each of the following differential equations:

(a) \( y'' + 10y' + 25 = 0 \)

(b) \( y'' + 10y' + 25y = 0 \)

(c) \( y'' + 10y' + 29y = 0 \)

(d) \( y'' + 10y' + 24y = 0 \)

Ex. 5. Solve the initial value problem \( y'' + y' - 2y = 2t, \) \( y(0) = 0, \) \( y'(0) = 1. \)

Ex. 6. Find a particular solution of the equation \( y'' + 3y = 3 \sin 2t. \)

Ex. 7. Given that \( y_1(x) = e^x \) is a solution of the ODE \((x - 1)y'' - xy' + y = 0, \) \( x > 0, \) use the method of reduction of order to find a second independent solution of this equation.
Ex. 8. Use the variation of parameters method to find a particular solution of the equation \( y'' + 4y' + 4y = t^{-2}e^{-2t}, \ t > 0. \) (No credit for the solution found by another method.)

Test #3 material

Ex. 9. Find the general solution for the following differential equations:

(a) \( y^{(8)} - 18y^{(4)} + 81y = 0 \)
(b) \( y^{(4)} - 4y'' = t^2 + e^t \)

Ex. 10. Use power series with \( x_0 = 1 \) to solve \( y'' - xy' - y = 0. \) Find the recurrence formula and use it to find the first two non-zero terms in each of two independent solutions.

Ex. 11. Use Laplace transforms to solve \( y'' + 3y' + 2y = 1, \ y(0) = 1, \ y'(0) = 0. \) Recall that \( \mathcal{L}[e^{at}] = \frac{1}{s-a} \) for \( s > a. \)

Test #4 material

Ex. 12. Use eigenvalues and eigenvectors to find the general solution of the given systems of differential equations. The solution must be expressed in terms of real-valued functions.

(a) \[ x' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} x \]

(b) \[ x' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} x \]

(c) \[ x' = \begin{pmatrix} 6 & -3 \\ 3 & 0 \end{pmatrix} x \]

Ex. 13. Solve the following boundary value problem or show that it does not have a solution. \( y'' + 4y = 0, \ y(0) = 0, \ y(\pi) = 0. \)

Ex. 14. Determine whether the method of separation of variables can be used to replace the partial differential equation \( u_{xx} + u_{xt} + u_t = 0 \) by a pair of ordinary differential equations. If so, find the ordinary differential equations. Do not solve them.

Ex. 15. Solve the heat equation: \( u_t = 9u_{xx}, \ u(0, t) = u(2, t) = 0, \ u(x, 0) = 13 \) for \( 0 < x < 2. \)