Determine the largest possible domain of a function

**Example (1)**: Determine the largest possible domain of \( f(x, y) = (\sqrt{2x} + \sqrt[3]{3y}) \).

**Solution**: Any real value of \( y \) can make \( \sqrt[3]{3y} \) meaningful, and so the domain for \( \sqrt[3]{3y} \) is the whole \( y \)-axis. Only non-negative real value of \( x \) can make \( \sqrt{2x} \) meaningful, and so the domain for \( \sqrt{2x} \) is the half line \([0, \infty)\). Combining these facts, we conclude that the domain of the function \( f(x, y) = (\sqrt{2x} + \sqrt[3]{3y}) \) is the half plane where \( x \geq 0 \), or in set notation: \( \{(x, y) : 0 \leq x < \infty \text{ and } -\infty < y < \infty}\). 

**Example (2)**: Determine the largest possible domain of \( f(x, y) = \frac{xy}{x^2 - y^2} \).

**Solution**: To avoid zero denominators, we must have \( x^2 - y^2 \neq 0 \). Since \( x^2 - y^2 = (x-y)(x+y) \), the domain of this function is the whole \( xy \)-plane with the two straight lines \( y = x \) and \( y = -x \) taken away.