Determine the equation of a straight line in the plane

Some Facts about lines
(1) For non vertical line $L$, if $L$ passes a point $(x_0, y_0)$ with slope $m$, then an equation of line $L$ is

$$y - y_0 = m(x - x_0).$$

(2) If a non vertical line $L$ passes through distinct points $(x_1, y_1)$ and $(x_2, y_2)$, then the slope of $L$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

(3) A vertical line passing through $(x_0, y_0)$ has equation $x = x_0$; a horizontal line passing through $(x_0, y_0)$ has equation $y = y_0$.

(4) A line $L$ intersects the $x$-axis at $(a, 0)$ (the number $a$ will be called the $x$-intersect of $L$) and the $y$-axis at $(0, b)$ (the number $b$ will be called the $y$-intersect of $L$) has equation

$$\frac{x}{a} + \frac{y}{b} = 1.$$

(5) Two non vertical lines are parallel if and only if they have the same slope; and two non vertical lines are perpendicular to each other if and only if the product of their slopes equals $-1$.

(6) The angle $\theta$ between the $x$-axis and a non vertical line $L$ is called an angle of inclination, and the slope of $L$ is $m = \tan \theta$.

Example 1 Find an equation of the line $L$ that passes through $(2, -3)$ and $(5, 3)$.

Solution: Then the slope of $L$ is

$$m = \frac{3 - (-3)}{5 - 2} = \frac{6}{3} = 2.$$

As $L$ passes through $(2, -3)$, an equation of $L$ is

$$y - (-3) = 2(x - 2).$$

Example 2 Find an equation of the line $L$ that has slope 6 and $y$-intersect 7.
Solution: The line has y-intersect 7 means that \((0, 7)\) is on the line. Therefore an equation of \(L\) is
\[
y - 7 = 6(x - 0).
\]

Example 3 Find an equation of the line \(L\) that has angle of inclination \(135^\circ\) and contains \((4, 2)\).

Solution: The slope of \(L\) is \(m = \tan 135^\circ = \tan \frac{3\pi}{4} = -1\) Therefore an equation of \(L\) is
\[
y - 2 = -1(x - 4).
\]

Example 4 Find an equation of the line \(L\) that passes through \((1, 5)\) and is parallel to the line with equation \(2x + y = 10\).

Solution: Rewrite \(2x + y = 10\) into \(y = -2x + 10\). Then the slope of this line is \(-2\). Note that \(L\) also has the same slope and so an equation of \(L\) is
\[
y - 5 = -2(x - 1).
\]

Example 5 Find an equation of the line \(L\) that passes through \((1, 5)\) and is perpendicular to the line with equation \(2x + y = 10\).

Solution: Rewrite \(2x + y = 10\) into \(y = -2x + 10\). Then the slope of this line is \(-2\). Let \(m\) denote the slope of \(L\). Then \((-2)m = -1\), and so \(m = \frac{1}{2}\). Thus an equation of \(L\) is
\[
y - 5 = \frac{1}{2}(x - 1).
\]