Find derivatives by using differentiation rules

**Differentiation Rules:**

(1) Derivative of a constant: Let \( C \) be a constant, then \( \frac{d}{dx} C = 0 \).

(2) Power Rule: For a real number \( n \),
\[
\frac{d}{dx} x^n = nx^{n-1}.
\]

(3) Linear Property: For constant \( a \) and \( b \) and functions \( f(x) \) and \( g(x) \),
\[
[af(x) + bg(x)]' = af'(x) + bg'(x) \quad \text{and} \quad [af(x) - bg(x)]' = af'(x) - bg'(x).
\]

(4) Product Rule: \[
(f(x)g(x))' = f'(x)g(x) + f(x)g'(x).
\]

(5) Quotient Rule: \[
\frac{f(x)}{g(x)}' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.
\]

(6) Generalized Power Rule: for a real number \( n \),
\[
\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x).
\]

**Example 1** Apply differentiation rules to find the derivative of \( f(x) = (2x^2 - 1)(x^3 + 2) \).

**Solution:** The function \( f(x) \) is a product, and each factor is a polynomial. So we first apply Product Rule, and then apply the linear property and the power rule to get:
\[
f'(x) = (2(2)x^{2-1} - 0)(x^3 + 2) + (2x^2 - 1)(3x^{3-1} + 0) = 4x^3 + 2 + 3x^2(2x^2 - 1) = 4x^4 + 8x + 6x^4 - 3x^2 = 10x^4 - 3x^2 + 8x.
\]

**Example 2** Apply differentiation rules to find the derivative of \( f(x) = \frac{2x^2 - 1}{x^3 + 2} \).

**Solution 1:** The function \( f(x) \) is a quotient. So we first apply Quotient Rule, and then apply the linear property and the generalized power rule to get:
\[
f'(x) = \frac{(2(2)x^{2-1} - 0)(x^3 + 2) - (2x^2 - 1)(3x^{3-1} + 0)}{(x^3 + 2)^2} = \frac{4x^3 + 2 + 3x^2(2x^2 - 1)}{(x^3 + 2)^2} = \frac{4x^4 + 8x + 6x^4 - 3x^2}{(x^3 + 2)^2} = \frac{x(10x^3 - 3x + 8)}{(x^3 + 2)^2}.
\]

**Solution 2:** View the function \( f(x) \) as a product by using negative exponents.
\[
f(x) = (2x^2 - 1)(x^3 + 2)^{-1}.
\]
Then apply Product Rule, and then the linear property and the power rule to get the answer (the answer is intentionally not simplified to make it easier for a reader to see the computation process).

\[
f'(x) = (2(2)x^{2-1} - 0)(x^3 + 2)^{-1} + (2x^2 - 1)(-1)(x^3 + 2)^{-1-1}(3x^3-1 + 0)
= 4x(x^3 + 2)^{-1} - 3x^2(2x^2 - 1)(x^3 + 2)^{-2}.
\]

**Example 3** Apply differentiation rules to find the derivative of \( f(x) = \frac{2x^3 - 3x^2 + 4x - 5}{x^2} \).

**Solution**: The function \( f(x) \) is a quotient. But we are not in a hurry to apply Quotient Rule, as we observed that the denominator is just a power of \( x \). Thus we first simplify the fraction.

\[
f(x) = \frac{2x^3 - 3x^2 + 4x - 5}{x^2} = \frac{2x^3}{x^2} - \frac{3x^2}{x^2} + \frac{4x}{x^2} - \frac{5}{x^2} = 2x - 3 + 4x^{-1} - 5x^{-2}.
\]

Then apply the linear property and the power rule.

\[
f'(x) = [2x - 3 + 4x^{-1} - 5x^{-2}]' = 2 - 0 + 4(-1)x^{-1-1} - 5(-2)x^{-2-1} = 2 - \frac{4}{x^2} + \frac{10}{x^3}.
\]

**Example 4** Write an equation of the line tangent to the curve \( f(x) = \left(\frac{2}{x} - \frac{1}{x^2}\right)^{-1} \) at the point \((2, 4/3)\).

**Solution**: The equation of this line has the form

\[
y - \frac{4}{3} = f'(2)(x - 2).
\]

To find the slope \( f'(2) \), we first compute the derivative \( f'(x) \). To do that it may be better to simplify the fraction first

\[
f(x) = \left(\frac{2}{x} - \frac{1}{x^2}\right)^{-1} = \left(\frac{2x}{x^2} - \frac{1}{x^2}\right)^{-1} = \left(\frac{2x - 1}{x^2}\right)^{-1} = \frac{x^2}{2x - 1}.
\]

Then, apply Quotient Rule

\[
f'(x) = \frac{2x(2x - 1) - 2x^2}{(2x - 1)^2} = \frac{4x^2 - 2x - 2x^2}{(2x - 1)^2} = \frac{2x^2 - 2x}{(2x - 1)^2}.
\]

Hence \( f'(2) = \frac{4}{9} \), and so the equation is

\[
y - \frac{4}{3} = \frac{4}{9}(x - 2).
\]