Math 156 9-24-2012 Worksheet

Name:

Instruction: For each problem, do the following steps:
(1) Sketch the graph if it is not given.
(2) Determine the integration bounds (x-bound? y-bound?) and the integrand (how do I compute the area of a cross-section?). Then set up the integral.
(3) Evaluate the integral, if it is instructed to do so.
(This is to train you to have the right habit to approach the problem. Skipping steps may result in little credit for the problem).

1: Set up an integral that compute the volume of a solid obtained by rotating the region bounded by \( y = x^3, y = x, x \geq 0 \) about the x-axis. (Do not compute the integral).

\[
V = \int_0^1 \pi \left( r_{\text{large}}^2 - r_{\text{small}}^2 \right) \, dx = \int_0^1 \pi (x^2 - x^6) \, dx
\]

2: Set up an integral that compute the volume of a solid obtained by rotating the region bounded by \( x = 2\sqrt{y}, x = 0, y = 9 \) about the y-axis. (Do not compute the integral).

\[
V = \int_0^9 \pi r^2 \, dy = \int_0^9 \pi (2\sqrt{y}) \, dy
\]

3: Find the volume of a solid obtained by rotating the region bounded by \( y = \frac{1}{x}, x = 1, x = 2, y = 0 \) about the x-axis.

\[
V = \int_1^2 \pi \left( \frac{1}{x} \right)^2 \, dx = \pi \int_1^2 x^{-2} \, dx = \pi \left[ \frac{x^{-2+1}}{-2+1} \right]_1^2 = -\pi \left( \frac{1}{2} - 1 \right) = -\pi \left( -\frac{1}{2} \right) = \frac{\pi}{2}
\]