Math 156 9-21-2012 Worksheet

Name:

**Instruction:** For each problem, do the following steps:
1. Sketch the graph if it is not given.
2. Determine the integration bounds (x-bounds? y-bounds?) and the integrand (y_{top} - y_{bot} or x_{right} - x_{left}). Then set up the integral.
3. Evaluate the integral, if it is instructed to do so.
(This is to train you to have the right habit to approach the problem. Skipping steps may result in little credit for the problem).

1: Find the area of the shaded region.

- **Step 1:** Sketch the graph (Given)
- **Step 2:** Determine \( y_{top} = 5x - x^2 \), \( y_{bot} = x \)
  
  Need \( x \) bounds. Solve \( \int y = 5x - x^2 \) for \( x \)
  \[
  x = 5x - x^2 \Rightarrow x(4-x) = 0, \quad x = 0, \quad x = 4.
  \]
  \( a = 0, \quad b = 4 \)
  
  **Step 3:**
  
  \[
  \text{Ans.} = \int_0^4 [(5x-x^2) - x] \, dx = \int_0^4 (4x-x^2) \, dx = 2x^2 - \frac{x^3}{3} \bigg|_0^4 = 32 - \frac{64}{3} = \frac{32}{3}
  \]

2: Find the area of the region bounded by these curves \( y = 12 - x^2 \) and \( y = x^2 - 6 \).

- **Step 1:** Graph \( y = 12 - x^2 \) (Given)
  
  **Step 2:** Determine \( y_{top} = 12 - x^2 \), \( y_{bot} = x^2 - 6 \)
  Need \( x \)-bounds. Solve \( \int y = 12 - x^2 \) for \( x \).
  \[
  12 - x^2 = x^2 - 6 \quad \text{or} \quad x^2 = 9, \quad x = 3
  \]
  \( a = -3, \quad b = 3 \)
  
  **Step 3:**
  
  \[
  \text{Ans.} = \int_{-3}^3 [(12-x^2) - (x^2-6)] \, dx = \int_{-3}^3 (18-2x^2) \, dx
  \]
  
  \[
  = 18x - \frac{2x^3}{3} \bigg|_{-3}^3 = (54 - \frac{54}{3}) - (-54 - \frac{-54}{3}) = \frac{72}{3} = 24
  \]

3: Set up an integral that computes the area of the region bounded by these curves \( x = 2y^2 \) and \( x = 4 + y^2 \). **Do Not** evaluate the integral.

- **Step 1:** Graph \( x = 2y^2 \) and \( x = 4 + y^2 \)
  
  **Step 2:** Determine \( x_{left} = 2y^2 \), \( x_{right} = 4 + y^2 \)
  Need \( y \)-bounds. Solve \( \int x = 2y^2 \) for \( y \)
  \[
  2y^2 = 4 + y^2, \quad \Rightarrow \quad y^2 = 4 \quad \Rightarrow \quad y = \pm 2 \quad \Rightarrow \quad a = -2, \quad b = 2
  \]
  
  **Step 3:**
  
  \[
  \text{Ans.} = \int_{-2}^2 [(4+y^2) - 2y^2] \, dy = \int_{-2}^2 (4-y^2) \, dy
  \]
  
  \[
  = 4y - \frac{y^3}{3} \bigg|_{-2}^2 = (8 - \frac{8}{3}) - (-8 - \frac{-8}{3}) = \frac{32}{3}
  \]