Important: Your should write your solutions in the space given. If you have to write your solution outside the space provided, you must indicated clearly where your solutions are. Failing to do so may result in losing credit points as grader will ignore solutions that are not written in the given space.

PART 1. This portion of the test consists of Fill in the Blank questions. Write your answers neatly and legibly in the spaces provided. Some partial credit may be given.

1. (3 points) The most general antiderivative of \( f(x) = 4x + 15x^4 \) is ______.

2. (3 points) The most general antiderivative of \( f(x) = \sec^2(x) + \sin(x) \) is ______.

3. (3 points) The function \( f(x) = x^3 + 2x \) is continuous on \([0, 2]\) and differentiable on \((0, 2)\). Then the number \( c \) inside \((0, 2)\) that satisfies the conclusion of the Mean Value Theorem is \( c = ______ \).

4. (3 points) The position of a moving particle at time \( t \) is \( s(t) \). If its velocity \( v(t) = 3t^2 - 1 \), and if \( s(0) = 4 \), then the position function \( s(t) \) is ________________.

5. (3 points) The value \( \lim_{x \to 0} \frac{e^{4x} - 1}{x} \) equals ______.

6. (6 points) Given \( f(x) = x^{\frac{1}{3}} - 3x \), then \( f'(x) = ______ \), and all its critical numbers are ________.
PART 2: This portion of the test consists of multiple choice problems. No partial credit is given in this section, so work very carefully. *Value: 3 points each*

7. The limit \( \lim_{x \to 1} \frac{\ln(x^2)}{x - 1} \) is equal to

   (A) 1  (B) −1  (C) 0  (D) 2  (E) −2  (F) None of These.

8. The limit \( \lim_{x \to 0} \frac{\cos(2x) - 1}{x^2} \) is equal to

   (A) 1  (B) −1  (C) 0  (D) 2  (E) −2  (F) None of These.

9. For the function \( f(x) = x + \frac{4}{x} \) on \([1, 3]\), which of the following is a correct statement?

   (A) \( f(x) \) has absolute maximum at \( x = 1 \) and absolute minimum at \( x = 2 \).
   (B) \( f(x) \) has absolute maximum at \( x = 1 \) and absolute minimum at \( x = 3 \).
   (C) \( f(x) \) has absolute maximum at \( x = 2 \) and absolute minimum at \( x = 1 \).
   (D) \( f(x) \) has absolute maximum at \( x = 2 \) and absolute minimum at \( x = 3 \).
   (E) \( f(x) \) has absolute maximum at \( x = 3 \) and absolute minimum at \( x = 1 \).
   (F) \( f(x) \) has absolute maximum at \( x = 3 \) and absolute minimum at \( x = 2 \).
   (G) None of these.

10. For a function \( f(x) = 2x^3 - 24x^2 \), which of the following is a correct statement?

    (A) \( f(x) \) is concave up on \((−∞, 2)\) and concave down on \((2, ∞)\).
    (B) \( f(x) \) is concave up on \((−∞, 4)\) and concave down on \((4, ∞)\).
    (C) \( f(x) \) is concave down on \((−∞, 2)\) and concave up \((2, ∞)\).
    (D) \( f(x) \) is concave up on \((−∞, 0)\) and concave down on \((0, ∞)\).
    (E) \( f(x) \) is concave down on \((−∞, 0)\) and concave up \((0, ∞)\).
    (F) \( f(x) \) is concave down on \((−∞, 4)\) and concave up \((4, ∞)\).
    (G) None of these.

11. The most general antiderivative of \( e^x - \sin x \) is equal to

    (A) \( e^x - \cos x + C \)  (B) \( e^x + \cos x + C \)  (C) \( e^{x+1} + \cos x + C \)
    (D) \( e^{x+1} - \cos x + C \)  (E) \( e^x + \sin x + C \)  (F) None of These.
12. The most general antiderivative \( \frac{x^3 - 1}{x^2} \) is equal to

(A) \( \frac{3x^2 - 1}{2x} + C \)  
(B) \( \frac{x^4 - x}{x^3} + C \)  
(C) 0

(D) \( \frac{x^2}{2} + \frac{1}{x} + C \)  
(E) \( \frac{x^2}{2} + \ln |x| + C \)  
(F) None of These.

13. Given two real numbers with distance 50 and minimum possible product, the bigger of the two numbers must be

(A) 50  
(B) 0  
(C) -25  
(D) -100  
(E) 25  
(F) None of These.

**PART 3:** This portion of the exam will be graded on a partial credit basis. **Answers without supporting work shown on the paper will receive NO credit.**

14. (10 points) Find the absolute maximum value and the absolute minimum value of \( f(x) = 3x^3 - x \) on the interval \([-1,0]\).
15. (5 points each) Find the most general antiderivative of each of the following functions.

(a) \( f(x) = 1 - 4x^3 + x^{\frac{4}{5}}. \)

(b) \( f(x) = \frac{1}{x} + \cos(x). \)

16. (4 points each) Find each of the limits.

(a) \( \lim_{x \to 1} \left( \frac{1}{x-1} - \frac{1}{\ln x} \right). \)

(b) \( \lim_{x \to 0^+} x^{2x}. \)
17. (10 points) A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?
18. Given a function \( f(x) = \frac{2(x^2 - 9)}{x^2 - 4} \), and \( f'(x) = \frac{20x}{(x^2 - 4)^2} \), and \( f''(x) = \frac{-20(3x^2 + 4)}{(x^2 - 4)^3} \), sketch the graph \( y = f(x) \) by completing each of the following.

(a) (5 points) Determine the domain, the value \( f(0) \) (the \( y \)-intercept), the vertical asymptote(s) and the horizontal asymptote(s) of \( y = f(x) \).

(b) (5 points) Determine the interval(s) where \( f(x) \) is increasing, the interval(s) where \( f(x) \) is decreasing, and determine the value of local maxima/local minima of \( f(y) \), if there are any.

(c) (5 points) Determine the interval(s) where \( f(x) \) is concave upward, and the interval(s) where \( f(x) \) is concave downward, and identify any inflection point(s), if there are any.
(d) (5 points) Sketch the graph $y = f(x)$ reflecting all the features above.
Math 155 Exam Grade Sheet

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