1. Related rates

(1.1) If the length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

(1.2) If \( x^2 + y^2 = 25 \) and \( \frac{dy}{dt} = 6 \), find \( \frac{dy}{dt} \) when \( y = 4 \).

(1.3) At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM of the same day?

(1.4) Water is leaking out of an inverted conical tank at a rate of 10,000 cm\(^3\)/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

2. Linear Approximation and differentials

(2.1) Find the linearization \( L(x) \) of the function \( f(x) = \frac{1}{\sqrt{2 + x}} \) at \( a = 2 \).

(2.2) Use linear approximation (or differentials) to estimate \( \sqrt{99.8} \) and \( 1/1002 \).

(2.3) Find the differential of the following.

(a) \( y = s/(1 + 2s) \).

(b) \( y = \sqrt{4 + 5x} \).

3. Exponential Functions

(3.1) Find the domain of these functions:

(a) \( f(x) = \frac{1}{1 - e^x} \)

(b) \( f(x) = \sqrt{1 - e^x} \)

(3.2) Find the limits:

(a) \( \lim_{x \to \infty} \frac{2 + 3^x}{4 - 3^x} \).

(b) \( \lim_{x \to -2^+} e^{3/(2-x)+x} \).
4. Inverse Functions and Logarithm

(4.1) If \( f \) is a one-to-one function such that \( f(3) = 4 \), what is \( f^{-1}(4) \)?

(4.3) Find a formula for the inverse function of each of the following functions.
(a) \( f(x) = \frac{4x - 1}{2x + 3} \).
(b) \( f(x) = \frac{1 + e^x}{1 - e^x} \).

(4.4) Find the exact value of the following expression without using a calculator.
\[ \log_6 \left( \frac{1}{30} \right), \log_8 2, \ln e^{\sqrt{2}}, \log_5 10 + \log_5 (20) - 3 \log_5 (2), 2^{\log_2 (3) + \log_2 (5)}. \]

5. Inverse Functions and Logarithm

(5.1) Find the derivative of these functions.
(a) \( f(x) = \ln(x^2 + 10) \).
(b) \( f(x) = \cos(\ln(x)) \).
(c) \( f(x) = \ln(\sqrt{x}) \).
(d) \( f(x) = \log_{10} \left( \frac{x}{x - 1} \right) \).
(e) \( f(x) = \ln(x^4 \sin^2(x)) \).
(f) \( f(x) = \ln \sqrt{\frac{3x + 2}{3x - 2}} \).
(g) \( f(x) = \sqrt{x}e^x \).
(h) \( f(x) = 10^{1-x^2} \).
(i) \( f(x) = e^{\tan(6x)} \).
(j) \( f(x) = 2^{3x^2} \).

(5.2) Find \( y'' \).
(a) \( y = \frac{\ln(x)}{x^2} \).
(b) \( y = \ln(\sec(x) + \tan(x)) \).

(5.3) Use logarithmic differentiation to find \( y' \) for each function below.
(a) \( y = \sqrt{x}e^{x^2}(x^2 + 1)^{10} \).
(b) \( y = x^{\cos(x)} \).
(c) \( y = (\sqrt{x})^{x^2} \).
(d) \( y = \sin(x)\ln(x) \).

(5.4) (Implicit differentiation)
(a) If \( e^{x^2}y = x + y \), find \( y' \).
(b) If \( x^y = y^x \), find \( y' \).
(c) Find an equation of the tangent line to the curve $xe^y + ye^x = 1$ at the point $(0,1)$.

6. Exponential Growth and Decay

(6.1) (Radioactive decay) Bismuth-210 has a half life of 5.0 days.
(a) A sample originally has a mass of 800mg. Find a formula for the mass remaining after $t$ days.
(b) Find the mass remaining after 30 days.
(c) When is the mass reduced to 1 mg.?

(6.2) (Population Growth) A bacteria culture grows with constant relative growth rate. After 2 hours there are 600 bacteria and after 8 hours the count is 75000.
(a) Find the initial population.
(b) Find an expression for the population after $t$ hours.
(c) Find the number of cells after 5 hours.
(d) Find the rate of growth after 5 hours.
(e) When will the population reach 200,000?

7. Inverse Trigonometric Functions

(7.1) Find the exact value of each expression.

$\arctan(-1), \csc^{-1}(2), \sec^{-1}(\sqrt{2}), \arcsin(1), \sec(\arctan(2)), \cos(2\sin^{-1}(\frac{5}{13}))$.

(7.2) Simplify the expressions:

$\tan(\sin^{-1}(x)), \sin(\tan^{-1}(x)), \csc(\arctan(2x))$.

(7.3) Find the derivative of the following functions. Simplify where possible.

(a) $y = \sqrt{\tan^{-1}(x)}$.
(b) $y = \sqrt{1 - x^2} \arcsin(x)$.
(c) $y = x(\arctan(x))$.
(d) $y = e^{\sec^{-1}(x)}$.
(e) $y = x \cot^{-1}(x) - \sqrt{1 - x^2}$.
(f) $y = \tan^{-1}(x - \sqrt{1 + x^2})$.
(g) $y = \tan^{-1}\left(\frac{x}{a}\right) + \ln \left(\frac{x - a}{x + a}\right)$.

(7.4) (Implicit differentiation)

(a) Find $y'$ if $\tan^{-1}(xy) = 1 + x^2 y$.
(b) Find $y'$ if $\cos^{-1}(xy^2) + \sin^{-1}(x) = 1$.

(7.5) Find the limits.

(a) $\lim_{x \to \infty} \arccos\left(\frac{1 + x^2}{1 + 2x^2}\right)$.
(b) \( \lim_{x \to 0^+} \tan^{-1}(\ln(x)) \).