Math 156 Fall 2009  Quiz 2 Solutions

Name:

1: Exercise 15, Section 6.1 Evaluate the integral \( \int t \sin(3t) \, dt \).

Solution:  Use integration by parts, and set \( u = t \) and \( dv = \sin(3t) \, dt \). Then \( v = \int dv = \int (\sin(3t)) \, dt = -\frac{\cos(3t)}{3} \). Use the by-parts technique,

\[
\int t \sin(3t) \, dt = -\frac{t \cos(3t)}{3} - \int \frac{-\cos(3t)}{3} \, dt \\
= -\frac{t \cos(3t)}{3} + \frac{\sin(3t)}{9} + C.
\]

2: Modified Exercise 9, Section 6.1 Evaluate the integral \( \int \ln(x^2 + 1) \, dx \).

Solution:  This is a simpler problem in applying integration by parts. It is simpler because there is only one way to decide \( u = \ln(x^2 + 1) \) and \( dv = dx \), and so we do not have to make choices. As \( dv = dx \), we have \( v = x \) and so the by-parts technique give us

\[
\int \ln(x^2 + 1) \, dx = x \ln(x^2 + 1) - \int x \cdot \frac{2x}{x^2 + 1} \, dx \quad \text{Chain Rule: } dv = (\ln(x^2 + 1))' = \frac{(x^2 + 1)'}{x^2 + 1}
\]

\[
= x \ln(x^2 + 1) - 2 \int \frac{x^2}{x^2 + 1} \, dx \\
= x \ln(x^2 + 1) - 2 \int (1 - \frac{1}{x^2 + 1}) \, dx \quad \text{Division: } x^2 = (x^2 + 1) \cdot 1 - 1
\]

\[
= x \ln(x^2 + 1) - 2x + 2 \tan^{-1}(x) + C.
\]

3: Exercise 9, Section 6.1, (not included in the quiz) Evaluate the integral \( \int \ln(2x + 1) \, dx \).

Solution:  This would require us to use both integration by parts as well as substitution technique, in addition to the use of division. Again, we have no choice but letting \( u = \ln(2x + 1) \) and \( dv = dx \), and so \( v = x \).

\[
\int \ln(2x + 1) \, dx = x \ln(2x + 1) - \int x \cdot \frac{2}{2x + 1} \, dx \quad \text{Chain Rule: } dv = (\ln(2x + 1))' = \frac{(2x + 1)'}{2x + 1}
\]

\[
= x \ln(2x + 1) - \int \frac{2x}{2x + 1} \, dx \\
= x \ln(2x + 1) - \int \left(1 - \frac{1}{2x + 1}\right) \, dx \quad \text{Division: } 2x = (2x + 1) \cdot 1 - 1
\]

\[
= x \ln(2x + 1) - \int dx + \int \frac{dx}{2x + 1} \quad \text{Substitution: use } w = 2x + 1
\]

\[
= x \ln(2x + 1) - x + \ln |2x + 1| + C.
\]