Math 420  Homework 6

1. Write a program based on computing divided differences one diagonal at a time as shown below:

\[
\begin{bmatrix}
  f(x_0) \\
  f(x_1)
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  f(x_0) \\
  f(x_1) \\
  f[x_0,x_1]
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  f(x_0) \\
  f(x_1) \\
  f(x_0,x_1) \\
  f[x_0,x_1,x_2]
\end{bmatrix}
\rightarrow \ldots
\]

You will need nested loops for this. Define the MATLAB function as follows:

function \( D = \text{divdiff2}(x,y) \)

Then use divdiff2 in place of divdiff and see whether you get the same answer.

2. Write a MATLAB function

function \( y = \text{cubeinterp}(x,xdata,ydata) \)

to do cubic tabular interpolation as discussed in the online example: You are given arrays \( xdata \) and \( ydata \), where the \( xdata \) is ordered, i.e. \( xdata(1)<xdata(2)<\ldots<xdata(n) \). Given a value of \( x \), determine an index \( k \) for which \( xdata(k)<x=xdata(k+1) \). Then interpolate the four points \((x_j,y_j), j = k-1,k,k+1,k+2\) with a cubic polynomial \( p_3 \) and output \( y = p_3(x) \) for the given \( x \).

Note that if \( x < x(2) \) or \( x > x(n-1) \) the cubic interpolation cannot be performed, and an error message should be output. Also, you may assume that \( x \) is at most a one-dimensional array, in which case the output \( y \) should be the same size as \( x \) (you will need to create a loop and do the calculation for each element of \( x \) individually).

When you have a working function, investigate as follows the error estimate for this method \( |e(h)| \leq \frac{3}{128} \max_{x_k \leq t \leq x_{k+2}} |f^{(4)}(t)|h^4 \) using as an example data from \( f(x) = \sin(\pi x/2) \) on \([0,1]\) with data spaced by \( h = 1 \) and \( h = 0.05 \) (include one datapoint to the left of 0 and one to the right of 1 so that the entire interval \([0,1]\) can be covered): calculate and plot the error for values of \( x \) in \([0,1]\) and compare with the plot of \( \frac{3}{128} f^{(4)}(x)h^4 \).

Comment on what you see.

3. Interpolate the following functions on the stated interval \([a,b]\) at equally spaced interpolation nodes of spacing \( \frac{b-a}{n} \) where \( n = 5,10,15 \), in each case plotting the interpolant, the original function, and the datapoints. Make a separate plot of the error \( f(x) - p(x) \). Comment as appropriate on what you see. You don’t need to write your own interpolation functions - use the ones we developed in class or modify them if you like.

   a) \( f(x) = \sin(x), \text{ on } [-\pi, \pi] \)
b) \( f(x) = |x| \) on \([-1, 1]\) 

c) \( f(x) = \frac{1}{1 + x^2} \) on \([-5, 5]\)