1. For each of the following numerical integration methods:
   - write a formula for the associated composite quadrature rule.
   - Each formula is exact for polynomials up to what degree?
   - For each formula give the order of the error: $e(h) = O(h^?)$.
   - Finally, apply each formula to the calculation of $\int_0^1 \sin(\pi x)\,dx$ in such a way that three function evaluations are used in each case.

   a) Trapezoid rule
   b) Midpoint rule
   c) Simpson’s rule

2. Develop as follows a derivative formula $f'(x_0) \approx Af(x_0) + Bf(x_0 + h) + Cf(x_0 + 3h)$:
   First use undetermined coefficients to develop a formula $f'(0) \approx Af(0) + Bf(1) + Cf(3)$ and then obtain the desired formula by scaling and translation. Find the leading term in the error expansion in powers of $h$ of the formula $f'(x_0) \approx Af(x_0) + Bf(x_0 + h) + Cf(x_0 + 3h)$ that you obtain.

3. Consider the problem of solving $e^x + x = 0$
   a) Find two starting values with which you could begin the bisection method and between which the root is guaranteed to lie. Explain.
   b) Write the iteration for Newton’s method and perform three iterations of Newton’s method starting with $x_0 = 0$.
   c) If the equation is rewritten as $x = -e^x$, the associated fixed-point iteration will converge. Why?
      Knowing the approximate location of the root, for instance as determined in part b), by about what factor will the error decrease with each iteration?
      Perform 10 iterations of the fixed point iteration, starting with $x_0 = 0$. Estimate about how many more iterations would be required to obtain an error of about $10^{-6}$. (there is more than one way to go about this, from rough estimates to sophisticated theory - anything sensible will do)
4. a) Develop an integration formula exact for quadratic polynomials that will approximate
\[ \int_{-1}^{1} f(x) \, dx \] using the function values \( f(-2), f(0), f(2) \).

b) Your formula in a) is exact for polynomials of what maximum degree? (Hint: Try \( x^3 \))

c) How should the formula in part a) be scaled so as to obtain a formula for \( \int_{-h}^{h} f(x) \, dx \)? Find
the leading term in the error expansion of the scaled formula in the form \( C f^{(2)}(0) h^3 \), where you
determine the values of the ?'s (they’re different) and the value of \( C \).

5. Suppose we have an approximate integration formula
\[ \int_{-1}^{1} f(x) \, dx \approx Af(-1) + Bf(0) + Cf^\prime(0) + Df(1) \] that uses information on \( f' \) as well as \( f \). How
would you scale this formula to obtain an approximate integration formula
\[ \int_{-h}^{h} f(x) \, dx \approx A f(-h) + B f(0) + C f^\prime(0) + D f(h) \] ? Note that you are not required to actually find
\( A, B, C, D \) in this problem. (Hint: Substitute \( x = hu \) in the second integral and apply the original
formula)