Solutions to Problem Set 16, Math 461, Spring 2010

Problem 1 (4B.2) Show using similar triangles that if Cevians $AP$, $BQ$ and $CR$ are parallel, then the Cevian product is trivial.

Solution We compute $AR/RB = y/(x+y)$, $BP/PC = x/y$, and $CQ/AQ = (x+y)/x$. Thus

$$\frac{AR}{RB} \frac{BP}{PC} \frac{CQ}{AQ} = \frac{y}{x} \frac{x+y}{x+y} \frac{x}{y} = 1.$$

Problem 2 (4B.3) Consider $\triangle ABC$ as in the figure. Cevians $CR$ and $BQ$ are concurrent with line $AT$, which is parallel to $BC$. If we view $AT$ as a Cevian of $\triangle ABC$ in the generalized sense, then we know that the corresponding Cevian product is $(AR/RB)(CQ/QA)$. Show without reference to Ceva’s theorem that this quantity is equal to 1.

Solution We compute $AR/RB = K_{ATB}/K_{BTC}$ and also $CQ/QA = K_{BTC}/K_{ATB}$. So the product is 1.