The 27th Cumberland Conference on Combinatorics, Graph Theory and Computing

Eberly College of Arts and Sciences

May 16-17, 2014

Sponsored by National Science Foundation and Department of Mathematics of WVU
All talks will be arranged at Oglebay Hall and Ming Hsieh Hall

**Friday Morning, May 16, 2014**

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# Friday Afternoon, May 16, 2014

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Gaps in eigenfunctions of Graphs                               |
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| 14:30-14:55  | Daniel Cranston  
Graphs with chi = Delta have big cliques                                  |
|              | Ping Yang  
Forbidden pairs for the existence of a spanning Halin subgraph                |
|              | Nenad Cakic  
q-analogue of generalized Stirling numbers                                   |
| 14:55-15:20  | Hao Li  
On the 3-hued coloring of claw-free graph                                   |
|              | Vaidy Sivaraman  
Mock-threshold graphs                                                      |
|              | Henry Gould  
Catalan Numbers and the Protean nature of binomial coefficient notation     |
| 15:20-15:40  | Coffee Break                                                            |
| 15:40-16:40  | Plenary Talk: Hal Kierstead  
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|              | **MHH Room 122**                                                        |
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Connectedness and Hamiltonicity of graphs on vertex colorings             |
|              | Michael Schroeder  
Latin Squares with disjoint subsquares                                       |
|              | Erik Westlund  
Directed Tree Decompositions of Cayley Digraphs                             |
| 17:05-17:30  | Songling Shan  
2-Factors in Edge Chromatic Critical Graphs with Large Maximum Degree     |
|              | Kerry Ojakian  
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| 18:30        | Dinner: No. 1 Super Buffet                                               |
### Saturday Morning, May 17, 2014

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Matchings, factors and edge-colorings of regular graphs |
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Nathan Graber  
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| 10:50-11:10| Coffee Break                                                           |
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| 12:10-13:30| Lunch                                                                  |
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| 14:30-14:55 | Elyse Yeager  
Disjoint Cycles and Equitable Coloring  
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| 14:30-14:55 | Dong Ye  
Spanning trees in cubic graphs  
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| 14:30-14:55 | Bernard Lidicky  
On the Tree Search Problem with Non-uniform Costs  
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| 14:55-15:20 | Jian Cheng  
AVD Edge Coloring of 2-degenerate Graphs  
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| 14:55-15:20 | Michael Santana  
A condition for spanning trees with leaf distance at least d  
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| 14:55-15:20 | Mudassir Shabbir  
On a Variant of Graph Searching Problem  
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| 15:00-15:20 | Coffee Break |
| 15:40-16:05 | Peter Johnson  
Coloring to forbid monochromatic translates  
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| 15:40-16:05 | Yehong Shao  
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| 16:05-16:30 | Ryan Hansen  
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| 16:05-16:30 | Garth Isaak  
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| 16:05-16:30 | Meng Zhang  
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| 16:30-16:55 | David Offner  
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| 16:30-16:55 | Yanting Liang  
On minimum balanced bipartitions of triangle-free graphs  
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| 16:30-16:55 | Xuechao Li  
Lower bounds on edges in edge critical graphs with fixed maximum degrees  
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| 16:55-17:20 | Luke Nelsen  
Covering 2-edge-colored graphs with a pair of cycles  
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| 16:55-17:20 | Keke Wang  
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| 16:55-17:20 | Arezoo Nazi-Ghameshlo  
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OGH Room 106 |
Abstract

• Friday, May 16th, 9:00-10:00, MHH 122  Plenary Talk

Poset-free Families and Poset Packing in the Boolean Lattice

Jerry Griggs
University of South Carolina

We consider the Boolean lattice $B_n$ of all subsets of $[n] := \{1, \ldots, n\}$, ordered by inclusion. Considerable effort has gone towards determining asymptotically the largest size $La(n, P)$ of a family of subsets of $[n]$ that contains no (weak) subposet $P$. Griggs and Lu conjecture that for any $P$, $La(n, P)$ is asymptotic to an integer times $\left(\frac{n}{\lfloor n/2 \rfloor}\right)$. We survey the progress on the conjecture, especially the current status when $P$ is the four-element diamond poset, which remains open despite considerable effort.

We next consider more generally what happens when we forbid a family $F$ of posets. This leads naturally to the problem of maximizing the number of pairwise unrelated copies of a poset $P$ in $B_n$. By Sperner’s Theorem, this maximum number is $\left(\frac{n}{\lfloor n/2 \rfloor}\right)$ when $P$ is a single element. More generally, when $P$ is a chain on $k$ elements, the answer is asymptotic to $\frac{1}{2^{k-1}}\left(\frac{n}{\lfloor n/2 \rfloor}\right)$, as $n \to \infty$, by a result of Griggs, Stahl, and Trotter. For general posets $P$ it is interesting that we can solve this new problem asymptotically: The maximum number of unrelated copies of $P$ is $\sim \frac{1}{c(P)}\left(\frac{n}{\lfloor n/2 \rfloor}\right)$, where the constant $c(P)$ is the size of the smallest convex closure over all embeddings of $P$ into the Boolean lattice.

This problem was independently proposed by Katona, and solved by Katona and Nagy.

This is Joint work with Andrew Dove.

• Friday, May 16th, 10:00-10:25, OGH 102

Minimal Winning Numbers in Misere Nim with Alliances

Annela Kelly
Bridgewater State University

In misere Nim for more than two alliances, our previous analysis concluded that if two uneven alliances play the game, the larger alliance will always win the game. This talk will investigate the optimal distribution of alliances to determine the smallest value for $K$ such that the larger alliance will win the game.
A graph $G$ containing a perfect matching is said to have the property $E(m,n)$ if given any two matchings $M$ and $N$ with $|M| = m$ and $|N| = n$ and such that $M \cap N = \emptyset$, then $G$ contains a perfect matching $F$ such that $M \subseteq F$ and $N \cap F = \emptyset$.

Generalizing a number of earlier results dealing with this concept, we present the following:

Theorem: Let $G$ be a 5-connected triangulation of a surface $\Sigma$ different from the sphere. Suppose that $V_0 \subseteq V(G)$ with $|V(G) - V_0|$ even and that $M$ and $N$ are matchings in $G - V_0$ of sizes $m$ and $n$ respectively such that $M \cap N = \emptyset$. Then if the pairwise distance between any two elements of $V_0 \cup M \cup N$ is at least 5 and the face-width of the embedding of $G$ in $\Sigma$ is sufficiently large, there is a perfect matching $F$ in $G - V_0$ containing $M$ such that $F \cap N = \emptyset$.

This is joint work with Ken Kawarabayashi and Kenta Ozeki.

Many graph classes are defined by the following paradigm: each vertex is assigned something measuring its size and something measuring its tolerance. If the combined sizes exceed the combined tolerances, then there is a conflict and the corresponding vertices are adjacent in a conflict-tolerance graph. This work considers path representations of graphs. Consider a host graph $H$. A path representation $[H : r : q]$ of a target graph $G$ is a labeling in which each vertex is assigned a unique path of length $r$ found in $H$ in such a way that if $uv$ is an edge in $E(G)$, then the $P_r$ assigned to $u$ and the $P_r$ assigned to $v$ have at least a $P_q$ in common. This study considers representations in which the host tree is the complete graph on $n$ vertices, $[K_n; r; q]$ which will be referred to as $P_r; q$-representations.

Keywords: intersection graphs, interval graphs, graph representations, minimal forbidden subgraphs, trees
Adversarial Coloring, Covering, and Domination Games
Chip Klostermeyer,
University of North Florida

We survey results from a family of "eternal" graph problems. Each problem is modeled as a two-player game played on a graph. One player, called the defender, must maintain a graph property, such as a proper coloring or a dominating set, against an adversary, who forces the defender to make changes to their configuration. Each game is played over an infinite number of rounds, or until the adversary wins the game. Eternal graph coloring, vertex covering, and domination problems are considered. For instance, the eternal domination number of graph G is the minimum k such that the defender is always able to maintain a dominating set with k vertices, where at most one vertex from the current dominating set can be "exchanged" with a neighbor in each round. Bounds on each of the eternal parameters are developed and compared with traditional graph parameters such chromatic number, independence number, and domination number. Several open problems are presented. This talk surveys joint work done with J. Goldwasser, G. MacGillivray, and C.M. Mynhardt.

A Characterization of Near Outer Planar Graphs
Tanya Lueder
LSUA/Louisiana Tech University

This talk focuses on graphs containing an edge whose removal results in an outer-planar graph. We present partial results towards the larger goal of describing the class of all such graphs in terms of a finite list of excluded graphs. Specifically, we give a complete description of those members of this list that are not 2-connected or do not contain a subdivision of a three-spoke wheel. We also show that no members of the list contain a five-spoke wheel.
When all minimal \( k \)-vertex separators induce complete or edgeless subgraphs
Terry McKee
Wright State University

Let \( D_k \) denote the class of graphs such that, for every independent set \( \{v_1, \ldots, v_i\} \) of vertices with \( 2 \leq i \leq k \), if \( S \) is an inclusion-minimal set of vertices whose deletion would leave \( v_1, \ldots, v_i \) in \( i \) separate connected components, then \( S \) induces a complete subgraph; also, let \( D = \bigcap_{k \geq 2} D_k \).

For instance, \( D_2 \) is the class of chordal graphs. Others of these classes—along with some of the modified classes when “complete” is replaced by “edgeless” or by “complete or edgeless”—have been characterized recently.

I shall give unified characterizations of all of these classes.

Some recent results on flow conjectures
Yezhou Wu
The University of Hong Kong

Tutte’s 3-Flow Conjecture states that every 4-edge-connected graph has nowhere-zero 3-flow and his 5-Flow Conjecture says every bridgeless graph has nowhere-zero 5-flow. Jaeger conjectured that every \( 4p \)-edge-connected graph has nowhere-zero \((2 + 1/p)\)-flow, which generalizes 3-Flow Conjecture and implies 5-Flow Conjecture. In 2014 Lovasz, Thomassen, Wu and Zhang (JCTB) proved every \( 6p \)-edge-connected graph has nowhere-zero \((2 + 1/p)\)-flow. This talk we will discuss some relative problems and recent results on those conjectures.

Limit Points of Graph Eigenvalues
Zhibo Chen
Pennsylvania State University

In this talk I will present some results on limit points of graph eigenvalues, which are mainly from my joint work with Fuji Zhang of Xiamen University.

The study of limit points of graph eigenvalues was initiated by A. J. Hoffman in 1972. J. B. Shearer (1989) and Dasong Cao and Hong Yuan (1993-95) did meaningful work in this area. More recently, we proved that every complex
number is a limit point of digraph eigenvalues and that every real number is a limit point of graph eigenvalues. Recently we used Ramsey’s Theorem to get a result on graph spectra, which enabled us to disprove a conjecture posted in a 1995 paper by Dasong Cao and Hong Yuan.

• Friday, May 16th, 11:10-11:35, OGH 110

Perfect Transitive Triangle Tilings in Oriented Graphs
Theodore Molla
University of Illinois at Urbana-Champaign

If $G$ is a graph, directed graph or oriented graph, a perfect $H$-tiling of $G$ is a spanning subgraph consisting entirely of disjoint copies of $H$. In 1963 Corradi and Hajnal proved that if $G$ is a graph on $n$ vertices, $n$ is divisible by 3 and $\delta(G) \geq 2n/3$, then $G$ contains a perfect triangle tiling. We will discuss an analogue of this result for oriented graphs.

For an oriented graph $G$ and any vertex $v$ in $V(G)$, let $d^0(v)$ be the minimum of the out-degree and in-degree of $v$. Define the minimum semidegree of $G$, denoted $\delta^0(G)$, to be the minimum of $d^0(v)$ over all vertices $v$. We will show that if $n$ is sufficiently large and divisible by 3, then every oriented graph $G$ on $n$ vertices such that $\delta^0(G) \geq 7n/18$ has a perfect transitive triangle tiling. This supports a conjecture of Treglown and is best possible: For any $n$ divisible by 3 there exists an oriented graph $G$ on $n$ vertices such that $\delta^0(G)$ equals $\lceil 7n/18 \rceil - 1$ and such that $G$ does not contain a perfect transitive triangle tiling.

This is joint work with Jozsef Balogh and Allan Lo.

• Friday, May 16th, 11:35-12:00, OGH 102

Group connectivity of Cayley graphs
Taoye Zhang
Penn State Worthington Scranton

Tutte conjectured that every 4-edge-connected graph admits a nowhere-zero 3-flow, and Jaeger conjectured that every 5-edge-connected graph is $Z_3$-connected. As a partial result of Tutte’s conjecture, Potocnik et al proved that every Cayley graph of degree at least 4 on an Abelian group admits a nowhere-zero 3-flow. We prove that every Cayley graph of degree at least 5 on an Abelian group is $Z_3$-connected.
Sign Patterns and Point–Hyperplane Configurations

Wei Gao
Georgia State University

A sign pattern (matrix) is a matrix whose entries are from the set \{+, -, 0\}. The minimum rank (respectively, rational minimum rank) of a sign pattern matrix \(A\) is the minimum of the ranks of the real (respectively, rational) matrices whose entries have signs equal to the corresponding entries of \(A\). A sign pattern \(A\) is said to be condensed if \(A\) has no zero row or column and no two rows or columns are identical or negatives of each other. In this talk, we establish a new direct connection between condensed \(m \times n\) sign patterns with minimum rank \(r\) and \(m\) point–\(n\) hyperplanes configurations in \(\mathbb{R}^{r-1}\), and we use this connection to prove that for every sign pattern with minimum rank 3, if each column has at most 2 zero entries, then the rational minimum rank is also 3. Furthermore, we construct the smallest known sign pattern whose minimum rank is 3 but whose rational minimum rank is greater than 3. We also obtain other results in this direction.

Embedding directed graphs in books

Jinko Kanno
Louisiana Tech University

In the literature, studying directed graphs \(D(G)\) led to surprising connections among different properties in graphs \(G\), such as between the maximal directed paths in \(D(G)\) and the chromatic number of \(G\). Embeddings of \(D(G)\) in a space have been much less studied than embeddings of \(G\). We consider a special space called an \(n\)-page book. For a non-negative integer \(n\), an \(n\)-page book or \(n\)-book \(B\) consists of a line \(L\) in 3-space \(\mathbb{R}^3\) (called a spine) together with \(n\) distinct half-planes (called pages) with \(L\) as their common boundary. The book-thickness \(bt(G)\) of \(G\) is the smallest \(n\) such that \(G\) has an \(n\)-book embedding such that the usual graph embedding conditions (two edges cross only at their endpoints) hold and such that every vertex belongs to \(L\), and each edge is embedded within \(L\) or else within at most one component of \(B - L\).

In this talk, we similarly define the oriented book-thickness \(obt(D(G))\) for directed graphs. Assume that \(L\) is an oriented line (such as the positive \(z\)-axis in \(\mathbb{R}^3\)). The oriented \(n\)-book embedding is defined by the same conditions as above plus the following restriction for embedding directed edges: all edges within \(L\) share the same orientation as \(L\)- or all have the opposite orientation as \(L\), all edges in a page containing the edge linking \(i\) and \(j\) share the same orientation
as $i$ and $j$ in $L$ - or all have the opposite orientation as $i$ and $j$ do in $L$. For a given graph $G$ or a given class $\mathcal{F}$ of graphs, we seek the upper bound $M(G)$ of $\text{obt}(D(G))$ for arbitrary orientations of $G$, and the class upper bound $M(F)$ of $\text{obt}(D(G))$ for arbitrary orientations of arbitrary graphs $G$ in the class $F$.

- **Friday, May 16th, 13:30-14:30, MHH 122**  
  **Plenary Talk**

  **Gaps in eigenfunctions of Graphs**  
  Fan Chung  
  University of California, San Diego

  We will examine the gaps and stretches in eigenfunctions in connection with the Cheeger inequalities and Harnack inequalities.

- **Friday, May 16th, 14:30-14:55, OGH 102**

  **Graphs with $\chi = \Delta$ have big cliques**  
  Daniel Cranston  
  Virginia Commonwealth University

  Brooks’ Theorem states that if a graph has $\Delta \geq 3$ and $\omega \leq \Delta$, then $\chi \leq \Delta$ (here $\Delta$ is the maximum degree and $\omega$ is the clique number). Borodin and Kostochka conjectured that if $\Delta \geq 9$ and $\omega \leq \Delta - 1$, then $\chi \leq \Delta - 1$. We show that if $\Delta \geq 13$ and $\omega \leq \Delta - 4$, then $\chi \leq \Delta - 1$.

- **Friday, May 16th, 14:30-14:55, OGH 106**

  **Forbidden pairs for the existence of a spanning Halin subgraph**  
  Ping Yang  
  Georgia State University

  Given a family $\mathcal{F} = \{H_1, H_2, \ldots, H_k\}$ of graphs, we say that a graph $G$ is $\mathcal{F}$-free if $G$ contains no induced subgraph isomorphic to any $H_i$, $i = 1, 2, \ldots, k$. In particular, if $\mathcal{F} = \{H\}$, we simple say $G$ is $H$-free. If $G$ is $\{H_1, H_2\}$-free, then $\{H_1, H_2\}$ is called a forbidden pair of $G$. A Halin graph is a plane graph $H = T \cup C$ consisting of a spanning tree $T$ with no vertices of degree 2 and a cycle $C$ induced by the leaves of the tree $T$. Bondy, in 1973, showed that Halin graph is Hamiltonian. In 1991, Bedrossian characterized all forbidden pairs for hamiltonian graphs. In this paper, we will talk some results about the forbidden pairs for existence of a spanning Halin subgraph in graph $G$. 

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• Friday, May 16th, 14:30-14:55, OGH 110

q-analogue of generalized Stirling numbers
Nenad Cakic
University of Belgrade

We define q-analogue of generalized Stirling numbers of the first and second kind. Also, we study a modified approach to these numbers via differential operators. Moreover, we investigate new explicit formulas of some numbers. Furthermore, some relations between these numbers and generalized q-harmonic numbers and many new combinatorial identities are derived.

• Friday, May 16th, 14:55-15:20, OGH 102

On the 3-hued coloring of claw-free graphs
Hao Li
Renmin University of China

For an integer \( r > 0 \), a \((k, r)\)-coloring of a graph \( G \) is a proper \( k \)-coloring of the vertices such that every vertex of degree \( i \) will be adjacent to vertices with at least \( \min\{i, r\} \) different colors. The smallest integer \( k \) for which a graph has a \((k, r)\)-coloring is the \( r \)-hued chromatic number \( \chi_r(G) \). In this paper, we give an upper bound of the 3-hued chromatic number of claw-free graphs and show that this bound is best possible.

• Friday, May 16th, 14:55-15:20, OGH 106

Mock-threshold graphs
Vaidy Sivaraman
SUNY (Binghamton)

Ever since the resolution of the Strong Perfect Graph Conjecture by Maria Chudnovsky, Neil Robertson, Paul Seymour, and Robin Thomas, the subject of induced subgraphs has attracted the attention of several researchers. A graph is said to be a threshold graph if its vertices can be assigned weights (real numbers) such that there is an edge between two vertices if and only if the sum of their weights crosses a fixed threshold (another real number). There is a simple forbidden induced subgraph characterization of threshold graphs. We relax the definition of a threshold graph to get a bigger class, one we call the class of mock-threshold graphs. We will discuss their place with respect to other well-known classes of graphs, and aim for a forbidden induced subgraph characterization of mock-threshold graphs. This is joint work with Richard Behr and Thomas Zaslavsky.
Leonard Euler, Johann Andreas von Segner, Nicolaus von Fuss and Eugene Catalan (and others) all studied the same numbers. 2014 is the 200th anniversary of the birth of Catalan. Catalan numbers appear in many combinatorial and graph theoretic studies. We discuss the Protean or ambiguous nature of the binomial coefficient notation with special emphasis on formulas for the Catalan numbers. The most popular notation for Catalan numbers is $C(n) = \binom{2n}{n}/(n+1)$ where $n \geq 0$. We exhibit nine different-looking binomial coefficient notations for these numbers and point out difficulties that arise when these notations are used in defining $C(n)$ when $n$ is 0 or $-1$. In particular we discuss an ambiguity in defining $C(-1) = -1/2$ and suggest an elegant way to justify it. We also relate these matters to the generalized Vandermonde convolution and discuss identities due to George Andrews, Tom Koshy and David Jonah.


Many graph algorithms depend on carefully preordering vertices. For instance, if graph $G = (V, E)$ has a vertex order so that every vertex is preceded by fewer than $k$ neighbors then First-Fit applied in this order yields a $k$-coloring of $G$. The coloring number of $G$ is the least $k$ for which such an order exists, i.e. one more than its degeneracy. For a fixed order, vertex $z$ is $t$-reachable from vertex $v$ if $z$ precedes $v$ and there is a path of length at most $t$ whose internal vertices succeed $v$. The $t$-coloring number of $G$ is the least $k$ for which there is an ordering of $V$ such that fewer than $k$ vertices are $t$-reachable from each vertex. The acyclic chromatic number is bounded in terms of 2-coloring number, but not coloring number.

In the $t$-coloring game two players Alice and Bob alternately choose unchosen vertices of $V$. This orders $V$—first chosen, first in order. The score is the least $k$ such that fewer than $k$ vertices are $t$-reachable from each vertex. The minimum score that Alice can force is the game $t$-coloring number of $G$. 
I will survey results and applications concerning (game) $t$-coloring numbers, including an application of game coloring to a non-game problem.

• Friday, May 16th, 16:40-17:05, OGH 102

Connectedness and Hamiltonicity of graphs on vertex colorings
Daniel McDonald
University of Illinois at Urbana-Champaign

Can we obtain any proper $k$-coloring of a graph $H$ from any other by hopping from proper $k$-coloring to proper $k$-coloring, with consecutive colorings only differing on small connected subgraphs? Let $G^j_k(H)$ be the graph whose vertices are the proper $k$-colorings of $H$, with an edge joining two colorings $c$ and $c'$ if $H$ has a connected subgraph on at most $j$ vertices containing all vertices where $c$ and $c'$ differ. Our question becomes that of whether $G^j_k(H)$ is connected.

Similarly, let a "Gray code" of proper $k$-colorings of $H$ be a cyclic ordering of all such colorings so that consecutive colorings agree outside some connected subgraph on at most $j$ vertices. A Gray code exists if and only if $G^j_k(H)$ is Hamiltonian. The connectedness and Hamiltonicity of $G^j_k(H)$ have been studied previously. We introduce and study the parameters $g_k(H)$ and $h_k(H)$, which denote the minimum $j$ such that $G^j_k(H)$ is connected or Hamiltonian, respectively.

• Friday, May 16th, 16:40-17:05, OGH 106

Latin Squares with disjoint subsquares
Michael Schroeder
Marshall University

In a latin square of size $n$, a subsquare of size $k$ is a collection of $k$ rows and $k$ columns whose intersecting cells consist of $k$ different symbols. Two subsquares are disjoint if their rows, columns, and symbols are each disjoint. A latin square realizing a partition $(n_1, n_2, \ldots, n_t)$ of $n$ is one with $t$ subsquares, each of size $n_1, n_2, \ldots, n_t$ respectively, which are pairwise disjoint.

We review some known results involving certain partitions, and improve some established bounds by Heinrich.

This is joint work with Jaromy Kuhl at UWF.
Directed Tree Decompositions of Cayley Digraphs
Erik Westlund
Kennesaw State University

In 1963, Ringel conjectured that the complete graph on \(2m + 1\) vertices can be decomposed by any tree with \(m\) edges. In the mid 1980s, Graham and Häggkvist conjectured more generally that every \(2m\)-regular graph and every \(m\)-regular bipartite graph can be decomposed by any tree with \(m\) edges. Fink showed in 1994 that for any directed tree \(T\), the directed Cayley graph \(X = DCay(G; S)\) is \(T\)-decomposable if \(|S| = |E(T)|\) and \(S\) is a minimal generating set of \(G\). Building upon that technique, we present an enlarged family of directed Cayley graphs that are \(T\)-decomposable by carefully relaxing the minimality condition on the connection set \(S\) using a simple concept from combinatorial group theory.

2-Factors in Edge Chromatic Critical Graphs with Large Maximum Degree
Songling Shan

In 1968, Vizing conjectured that, if \(G\) is an \(n\)-vertex edge chromatic critical graph with \(\chi'(G) = \Delta(G) + 1\), then \(G\) contains a 2-factor. We verify this conjecture for \(n \leq 2\Delta(G)\).
This is joint work with Guantao Chen.

Relating graphs to factorizations of permutations
Kerry Ojakian
BCC (CUNY)

Consider the symmetric group on \(n\) letters. Denes (1959) counted the number of factorizations of the cycle \((12...n)\) into \((n - 1)\) transpositions, using the known count of the number of vertex labeled trees on \(n\) vertices. Later, others found explicit bijections between the factorizations and the trees; Goulden and Yong (2002) found a “structural” bijection that preserves certain properties of the factorizations and the trees. Their construction uses the topological dual of a planar graph. We provide an alternative proof which uses a “combinatorial dual” of a graph. I will discuss aspects of this proof, the tools we develop, and how they might be used to deal with graphs beyond trees, and permutations beyond the cycle. This is joint work with Nikos Apostolakis.
One Open Problem of Paul Erdős is Closed: The Number of Maximal Triangle-free Graphs on n Vertices

Rupei Xu
The University of Texas at Dallas

Among the collections of open problems of Paul Erdős on Graph Theory on the website of Professor Fan Chung, there is one called "number of triangle-free graphs". Erdős, Kleitman and Rothschild proved that almost all triangle-free graphs on \( n \) vertices are bipartite. A triangle-free graph \( G \) is said to be maximal if adding any edge to \( G \) will result in a triangle. A problem on maximal triangle-free graphs is the following: Determine or estimate the number of maximal triangle-free graphs on \( n \) vertices. All graphs considered here are undirected, without loops or multiple edges, and with labelled vertices. In this talk, we give our solution of this problem and new progress on related open problems. This is joint work with Brendan McKay and Douglas Zare.

Hamilton cycle embeddings of graphs

Mark Ellingham
Vanderbilt University

A hamilton cycle embedding of a graph \( G \) is an embedding of \( G \) in a surface so that the boundary of every face is a hamilton cycle in \( G \). For such an embedding to exist, \( G \) must be \( r \)-regular for some \( r \). Hamilton cycle embeddings turn out to provide minimum genus embeddings for large families of graphs, and to illuminate the structure of minimum genus embeddings of complete bipartite graphs \( K_{r,s} \). In this talk we survey what is known about hamilton cycle embeddings. Many of the results discussed are from joint projects with Justin Schroeder, Chris Stephens, Adam Weaver and Xiaoya Zha.

Classification of Fulkerson Covering

Xiaofeng Wang
Indiana University Northwest

It was conjectured by Fulkerson that every bridgeless cubic graph has a collection of six perfect matchings that together cover every edge of the graph exactly twice. We study the different types of Fulkerson coverings and try to give a classification of it.
2-factors with k cycles in Hamiltonian graphs

Louis DeBiasio
Miami University

Bondy proved that if $G$ has minimum degree greater than $n/2$, then $G$ is pancyclic. Brandt, Chen, Faudree, Gould, and Lesniak proved that if $G$ is a graph on $n$ vertices with minimum degree at least $n/2$, then $G$ contains a 2-factor with exactly $k$ cycles (where $k$ is at most $n/4$). Moreover, both of these results are best possible.

However, regarding Bondy’s result, Amar, Flandrin, Fournier, and Germa showed that if $G$ is assumed to be Hamiltonian, then a smaller minimum degree is needed to guarantee that $G$ is pancyclic. Motivated by this, Faudree, Gould, Jacobson, Lesniak, and Saito asked if an analogous phenomenon occurred in the 2-factor with exactly $k$ cycles problem.

This question was answered in the affirmative by Sárközy, who used the regularity–blow-up method to show that $(1/2 - \epsilon)n$ is sufficient for some tiny $\epsilon > 0$.

I will discuss an improvement on this result, in which we give an elementary proof to show that $(2/5 + \epsilon)n$ is sufficient.

Joint work with Mike Ferrara and Tim Morris.

On the Type(s) of Minimum Size Subspace Partitions

Esmeralda Nastase
Xavier University

Let $V = V(kt+r, q)$ be a vector space of dimension $kt+r$ over the finite field with $q$ elements. Let $\sigma_q(kt+r,t)$ denote the minimum size of a subspace partition $\mathcal{P}$ of $V$ in which $t$ is the largest dimension of a subspace. We denote by $n_{d_i}$ the number of subspaces of dimension $d_i$ that occur in $\mathcal{P}$ and we say $[d_1^{n_{d_1}}, \ldots, d_m^{n_{d_m}}]$ is the type of $\mathcal{P}$. We show that a partition of minimum size has a unique partition type if $t+r$ is an even integer and we give partial results for the more intricate case when $t+r$ is an odd integer.
Saturday, May 17th, 10:25-10:50, OGH 102

Matchings, factors and edge-colorings of regular graphs

Vahan Mkrtchyan
West Virginia University

I will present some results on matchings, factors and edge-colorings of cubic and regular graphs. The results are mostly due to my students and myself. I will also present some new research problems over the topic.

Saturday, May 17th, 10:25-10:50, OGH 106

Linear Turan numbers of Linear Cycles

Nathan Graber
Miami University

An $r$-uniform hypergraph is called an $r$-graph. A hypergraph is linear if every two edges intersect in at most one vertex. Given a linear $r$-graph $H$ and a positive integer $n$, the linear Turan number $ex_L(n, H)$ is the maximum number of edges in a linear $r$-graph that does not contain $H$ as a subgraph.

An $r$-uniform linear cycle of length $k$, denoted by $C_k^r$, is a linear $r$-graph consisting of $k$ edges $E_1, \ldots, E_k$ such that $E_i$ and $E_j$ intersect if and only if $j = i + 1$ or $i = j + 1$ modulo $k$.

We show that for all $r \geq 3$, $m \geq 2$, there exist constants $c_{m,r}$ and $c'_{m,r}$ depending only on $m$ and $r$ such that $ex_L(n, C_{2m}^r) \leq c_{m,n} n^{1+1/m}$ and $ex_L(n, C_{2m+1}^r) \leq c'_{m,n} n^{1+1/m}$. This answers a question of Kostochka, Mubayi, and Verstraete.

For even cycles, our result extends the Bondy-Simonovits theorem on even cycles to linear hypergraphs.

Our results also allow us to establish nontrivial bounds on the hypergraph Ramsey numbers of linear cycles versus complete $r$-graphs.

This is joint work with T. Jiang and C. Collier-Cartaino.

Saturday, May 17th, 10:25-10:50, OGH 110

A New Proof for a Result of Kingan and Lemos

Jesse Williams
Wright State University

The prism graph is the planar dual of $K_5\setminus e$. Kingan and Lemos proved a decomposition theorem for the class of binary matroids with no Prism minor. In this paper, we present a different proof using fundamental graphs and blocking sequences.
A matroid is 3-connected if it does not break up as a 1-sum or a 2-sum. Numerous problems for matroids reduce easily to the study of 3-connected matroids. Two powerful inductive tools for dealing with 3-connected matroids are Tutte’s Wheels-and-Whirls Theorem and Seymour’s Splitter Theorem. For several years, Carolyn Chun, Dillon Mayhew, and I have been seeking analogues of these theorems for internally 4-connected binary matroids, that is, binary matroids that do not break up as a 1-, 2-, or 3-sum. The class of such matroids includes the cycle matroids of internally 4-connected graphs, those 3-connected simple graphs that are 4-connected except for the possible presence of degree-3 vertices. This talk will report on our progress towards finding these analogues.

Let \( f \) be an ordering of the edges of \( K_n \). An increasing path is a simple path (visiting each vertex at most once) such that the labels increase on successive edges. Let \( S(f) \) be the number of edges in the longest increasing path. Chvátal and Komlós raised the question of estimating \( m(n) \): the minimum value of \( S(f) \) over all orderings \( f \) of \( K_n \). The best known bounds on \( m(n) \) are between about \( \sqrt{n} \) and about \( n/2 \), due respectively to Graham and Kleitman, and to Calderbank, Chung, and Sturtevant. Although the problem is natural, it has seen essentially no progress for three decades.

In this paper, we consider the average case, when the ordering is chosen uniformly at random. We discover the surprising result that in the random setting, \( S(f) \) often takes its maximum possible value of \( n - 1 \) (visiting all of the vertices with a Hamiltonian increasing path). We prove that this occurs with probability at least about \( 1/e \). We also prove that with probability \( 1 - o(1) \), there is an increasing path of length at least \( 0.85n \), suggesting that this Hamiltonian (or near-Hamiltonian) phenomenon may hold asymptotically almost surely.

Joint work with Misha Lavrov.
Disjoint Cycles and Equitable Coloring
Elyse Yeager
University of Illinois at Urbana-Champaign

In 1963, Corrádi and Hajnal famously proved the following: if a graph $G$ has at least $3k$ vertices and minimum degree at least $2k$, then $G$ contains a set of $k$ vertex-disjoint cycles. The degree bound is sharp, but recently the theorem has been improved by considering Ore-type conditions. That is, by bounding the minimum degree sum of nonadjacent vertices, rather than the minimum degree.

An equitable coloring of a graph is a proper vertex coloring where no two color classes differ in size by more than one. When $|G| = 3k$, $G$ contains a set of $k$ disjoint cycles if and only if the complement of $G$ is equitably $k$-colorable. Chen, Lih, and Wu conjectured in 1994 that a connected graph $G$ is $\Delta(G)$-equitably colorable if it is different from $K_m, C_{2m+1},$ and $K_{2m+1,2m+1}$ for every $m \geq 1$. We discuss an Ore-type analog to this conjecture: that every $k$-colorable graph $G$ with maximum degree sum of adjacent vertices at most $2k + 1$ is equitably $k$-colorable unless it contains $K_{c,2k-c} + K_k$ for some odd $c$, $K_{1,2k} + K_{k-1}$, or a third graph in the case $k = 3$.

Spanning trees in cubic graphs
Dong Ye
Middle Tennessee State University

It is conjectured by Hoffman-Ostenhof that every cubic graph $G$ has a spanning tree $T$ such that every component of $G - E(T)$ is regular, i.e. it is an isolated vertex, a copy of $K_2$, or a cycle. We prove the conjecture for 3-connected plane cubic graphs, and our proof provides a polynomial time algorithm to find the spanning tree. This is joint work with Kenta Ozeki.

On the Tree Search Problem with Non-uniform Costs
Bernard Lidicky
University of Illinois at Urbana-Champaign

Searching in partially ordered structures has been considered in the context of information retrieval and efficient tree-like indexes, as well as in hierarchy based knowledge representation. In this talk we focus on tree-like partial orders and
consider the problem of identifying an initially unknown vertex in a tree by asking edge queries: an edge query $e$ returns the component of $T - e$ containing the vertex sought for, while incurring some known cost $c(e)$.

The Tree Search Problem with Non-Uniform Cost is: given a tree $T$ where each edge has an associated cost, construct a strategy that minimizes the total cost of the identification in the worst case.

Finding the strategy guaranteeing the minimum possible cost is an NP-complete problem already for input tree of degree 3 or diameter 6. The best known approximation guarantee is the $O(\log n/(\log \log n))$-approximation algorithm of [Cicalese et al. TCS 2012].

We improve upon the above results both from the algorithmic and the computational complexity point of view: We provide a novel algorithm that provides an $O(\log n/(\log \log n))$-approximation of the cost of the optimal strategy. In addition, we show that finding an optimal strategy is NP-complete even when the input tree is a spider, i.e., at most one vertex has degree larger than 2.

- **Saturday, May 17th, 14:55-15:20, OGH 102**

**AVD Edge Coloring of 2-degenerate Graphs**

Jian Cheng  
West Virginia University

The adjacent vertex-distinguishing chromatic index $\chi'_{avd}$ of a graph $G$ is the smallest integer $k$ for which $G$ admits a proper edge $k$-coloring such that no pair of adjacent vertices are incident to the same set of colors. We prove that if $G$ is a 2-degenerate graph, without isolated edges, then $\chi'_{avd} \leq \max\{6, \Delta(G) + 1\}$. Moreover, we also show that when $\Delta \geq 6$, $\chi'_{avd} = \Delta(G) + 1$ if and only if $G$ contains two adjacent vertices of maximum degree.

- **Saturday, May 17th, 14:55-15:20, OGH 106**

**A condition for spanning trees with leaf distance at least $d$**

Michael Santana  
University of Illinois at Urbana-Champaign

Let $i(G)$ denote the number of isolated vertices in a graph $G$. We will say that a tree $T$ has leaf distance at least $d$ if the distance between every pair of leaves in $T$ is at least $d$. In 2001, Kaneko conjectured the following: Let $d \geq 3$, and let $G$ be a connected, $n$-vertex graph with $n \geq d + 1$. If $i(G - S) < \frac{2}{d-2}|S|$ for all nonempty $S \subseteq V(G)$, then $G$ has a spanning tree with leaf distance at least $d$.

We translate this conjecture into the language of neighborhood unions and prove the conjecture for $d \geq n/3$. In particular, we prove a stronger version of the
conjecture with regards to these large values of $d$, requiring the above inequality only in certain cases. In this talk we will present our methods, a more refined sharpness example, and conclude with several open problems in regards to our new result.

This is joint work with Catherine Erbes (CU Denver), Theo Molla (UIUC), and Sarah Mousley (UIUC).

**Saturday, May 17th, 14:55-15:20, OGH 110**

On a Variant of Graph Searching Problem

Mudassir Shabbir

Rutgers University

Faults and viruses often spread in networked environments by propagating from site to neighboring site. We model this process of network contamination by graphs. Consider a graph $G = (V, E)$, whose vertex set is contaminated and our goal is to decontaminate the set $V(G)$ using mobile decontamination agents that traverse along the edge set of $G$. Temporal immunity $\tau(G) \geq 0$ is defined as the time that a decontaminated vertex of $G$ can remain continuously exposed to some contaminated neighbor without getting infected itself. The immunity number of $G$, $\iota_k(G)$, is the least $\tau$ that is required to decontaminate $G$ using $k$ agents. We study immunity number for some classes of graphs corresponding to network topologies and present upper bounds on $\iota_1(G)$, in some cases with matching lower bounds. Variations of this problem have been extensively studied in literature, but proposed algorithms have been restricted to monotone strategies, where a vertex, once decontaminated, may not be recontaminated. We exploit nonmonotonicity to give bounds which are strictly better than those derived using monotone strategies.

**Saturday, May 17th, 15:40-16:05, OGH 102**

Coloring to forbid monochromatic translates

Peter Johnson

Auburn University

For which planar triples (3-sets) is there a 2-coloring of the plane such that no translate in the plane of either triple is monochromatic? If, like most, you haven’t thought much about this question, you might be surprised to learn that it boils down to the same question with “planar” replaced by ”integer” and ”plane” replaced by ”integers”.

Recent joint work on this and related problems with Loren Anderson will be discussed, briefly.
Maximum Degree and the longest divalent path
Yehong Shao
Ohio University Southern

Let $G$ be a graph and $s \geq 0$ be an integer. If the graph $G$ has a certain property $P$ after deleting any $s$ vertices from $G$, then we say $G$ has a $s$-$P$ property. Let $l(G) = l$ be the length of the longest path whose internal vertices have degree 2 in $G$ such that if $l = 2$, then the path is not contained in a 3-cycle. Let $L^k(G)$ be the $k$-th iterated line graph of $G$. We show that if $L^{l+f(s)}(G)$ has a $s$-$P$ property, then $L^{|V(G)|-\Delta(G)+f(s)}(G)$ has a $s$-$P$ property, where $f(s)$ is a function of $s$. For certain properties, $L^{|V(G)|-\Delta(G)+f(s)}(G)$ has a $s$-$P$ property.

A New Minimum Spanning Tree Verification Algorithm for Graphs with Distinct Edge Weights
Matt Williamson
West Virginia University Institute of Technology

We present a new algorithm for the minimum spanning tree verification (MSTV) problem for undirected graphs. Our algorithm combines the insights of Boros;vkas algorithm for constructing a minimum spanning tree with a fast single source shortest path algorithm to efficiently identify edges which cannot be part of any minimum spanning tree. On a graph with $n$ vertices, $m$ edges, and $K$ distinct edge weights, our algorithm runs in $O(m + nK)$ time. It follows that our algorithm runs in linear time if $K$ is a fixed constant. We contrast the performance of our algorithm with Torben Hagerup’s linear time MSTV algorithm. Our experiments indicate that our algorithm is superior to Hagerup’s algorithm when $K \leq 24$.

Maximum number of copies of an induced subgraph in the $n$-cube
Ryan Hansen
West Virginia University

Let $Q_n$ denote the $n$-dimensional hypercube. Let $G$ be a vertex-induced embedded subgraph of $Q_d$. We call such a $G$ a configuration in $Q_d$. Let $f(G, d, n)$
be the maximum number of \( Q_d \) subgraphs of \( Q_n \) which have \( G \) embedded as a vertex-induced subgraph. An averaging argument shows that the limit

\[
\lim_{n \to \infty} \frac{f(G,d,n)}{\binom{n}{d}} = I(G,d)
\]

exists and we call \( I(G,d) \) the indicibility of \( G \) embedded in \( Q_d \).

Let \( M \) be the configuration in \( Q_3 \) given by the set of 4 vertices which induce two (parallel) edges. We provide a proof \( I(M,3) \). We also present a configuration \( F \) in \( Q_4 \) which leads to a 4-graph analog of a famous Turán 3-graph problem. We proved this analog which provides an upper bound on \( I(F,4) \) which matches a lower bound construction.

- **Saturday, May 17th, 16:05-16:30, OGH 106**

  **Min-max Theorems for the k-Path Partition Problem**
  
  Garth Isaak
  
  Lehigh University

  The 1-Hamiltonian Path problem is to test if there is a Hamiltonian path with a given vertex as one of the ends. This generalizes to the \( k \)-Path partition problem, which is to find a minimum partition into paths where \( k \) given vertices must be ends of paths. There are efficient algorithms for this problem on several structured graph classes. For several of the simplest of these classes, unit interval graphs, threshold graphs, block graphs, we present min-max characterization theorems. These results are surprisingly complex given the elementary structure of these classes.

- **Saturday, May 17th, 16:05-16:30, OGH 110**

  **Supereulerian Graph with Matching Number**
  
  Meng Zhang
  
  West Virginia University

Coloring 2-dimensional cubes in the hypercube so larger subcubes are polychromatic

David Offner
Westminster College

Denote the $n$ dimensional hypercube by $Q_n$. Given a graph $G$ which is a subgraph of the hypercube, call an edge coloring of the hypercube with a finite number of colors $G$-polychromatic if every copy of $G$ contains every color. Denote by $p(G)$ the polychromatic number of $G$, which is defined to be the largest number of colors such that there exists a $G$-polychromatic $r$-coloring of the edges of any hypercube. Alon, Krech and Szabo introduced polychromatic colorings in 2007 and proposed a generalization where larger subgraphs of the hypercube are colored. In this talk we survey what is known about polychromatic colorings and consider the following question: Given $d \geq 3$, we define $p^2(Q_d)$ to be the largest number of colors so that it is possible to color the copies of $Q_2$ in any hypercube so every copy of $Q_d$ contains a copy of $Q_2$ of each color. We show that $p^2(Q_3) = 3$ and give the best known bounds for other values of $d$.

This is joint work with Maria Axenovich, John Goldwasser, Bernard Lidicky, Ryan Martin, John Talbot, and Michael Young.

On minimum balanced bipartitions of triangle-free graphs

Yanting Liang
University of Wisconsin-Fond du Lac

A balanced bipartition of a graph $G$ is a partition of $V(G)$ into two subsets $V_1$ and $V_2$ that differ in cardinality by at most 1. A minimum balanced bipartition of $G$ is a balanced bipartition $V_1, V_2$ of $G$ minimizing $e(V_1, V_2)$, where $e(V_1, V_2)$ is the number of edges joining $V_1$ and $V_2$ and is usually referred to as the size of the bipartition. In this paper, we show that every 2-connected graph $G$ admits a balanced bipartition $V_1, V_2$ such that the subgraphs of $G$ induced by $V_1$ and by $V_2$ are both connected. This yields a good upper bound to the size of minimum balanced bipartition of sparse graphs. We also present two upper bounds to the size of minimum balanced bipartitions of triangle-free graphs.
• Saturday, May 17th, 16:30-16:55, OGH 110

Lower bounds on edges in edge critical graphs with fixed maximum degrees
Xuechao Li
The University of Georgia

We provide new adjacency lemmas for edge-chromatic critical graphs.

• Saturday, May 17th, 16:55-17:20, OGH 102

Covering 2-edge-colored graphs with a pair of cycles
Luke Nelsen
Miami University

Lehel conjectured that in every 2-coloring of the edges of $K_n$, there is a vertex disjoint red and blue cycle which span $V(K_n)$. Luczak, Rödl, and Szemerédi proved Lehel’s conjecture for large $n$ and Allen gave a different proof for large $n$. Bessy and Thomassé gave a proof for all $n$. BalOGH , Barát, Gerbner, Gyárfás, and Sárközy proposed a strengthening of Lehel’s conjecture where $K_n$ is replaced by any graph $G$ having minimum degree at least $3n/4$, and they proved an approximate version of their conjecture. We prove that their conjecture holds for sufficiently large $n$, and provide a small example which shows the result cannot be extended to all $n$.

(Joint work with Louis DeBiasio)

• Saturday, May 17th, 16:55-17:20, OGH 106

Hamilton cycles in 3-connected claw-free and net-free graphs
Keke Wang
West Virginia University

For an integer $s_1, s_2, s_3 > 0$, let $N_{s_1,s_2,s_3}$ denote the graph obtained by identifying each vertex of a $K_3$ with an end vertex of three disjoint paths $P_{s_1+1}$, $P_{s_2+1}$, $P_{s_3+1}$ of length $s_1$, $s_2$, and $s_3$, respectively. We determine a family $\mathcal{F}$ of graphs such that, every 3-connected $(K_{1,3}, N_{s_1,s_2,1})$-free graph $\Gamma$ with $s_1 + s_2 + 1 \leq 10$ is hamiltonian if and only if the closure of $\Gamma$ is $L(G)$ for some graph $G \notin \mathcal{F}$. We also obtain the following results.

(i) Every 3-connected $(K_{1,3}, N_{s_1,s_2,s_3})$-free graph with $s_1 + s_2 + s_3 \leq 9$ is hamiltonian.

(ii) If $G$ is a 3-connected $(K_{1,3}, N_{s_1,s_2,0})$-free graph with $s_1 + s_2 \leq 9$, then $G$ is hamiltonian if and only if the closure of $G$ is not the line graph of a member in $\mathcal{F}$.

(iii) Every 3-connected $(K_{1,3}, N_{s_1,s_2,0})$-free graph with $s_1 + s_2 \leq 8$ is hamiltonian.
Signed edge domination numbers of complete tripartite graphs

Arezoo Nazi-Ghameshlou
University of Tehran

The closed neighborhood $N_G[e]$ of an edge $e$ in a graph $G$ is the set consisting of $e$ and of all edges having an end-vertex in common with $e$. Let $f$ be a function on $E(G)$, the edge set of $G$, into the set $\{-1, 1\}$. If $\sum_{x \in N_G[e]} f(x) \geq 1$ for each edge $e \in E(G)$, then $f$ is called a signed edge domination function (SEDF) of $G$. The signed edge domination number $\gamma'_s(G)$ of $G$ is defined as $\gamma'_s(G) = \min\{\sum_{e \in E(G)} f(e) \mid f \text{ is an SEDF of } G\}$. In this presentation, we find the signed edge domination number for the complete tripartite graph $K_{m,n,p}$, where $m \leq n \leq p$ and $p \leq m + n$. 