Rational Expressions

- Section P.4
  - Rational comes from the Latin stem *ration* meaning thinking, reckoning, relating. A rational expression relates two polynomial functions by dividing them.

Application: Laffer Curve

- Economist Arthur Laffer developed a model for government revenue *r* in millions based on an increase in the percent of tax rate *t*. Is the model a rational expression (click to see model?)

$$r = \frac{1000t - 10t^2}{50 + t}$$

Rational Expression

- Let p and q be polynomial expressions, then a rational expression has the form

$$\frac{p}{q}$$

$$So\ r = \frac{1000t - 10t^2}{50 + t}\ is\ rational$$

Skill

- Determine which of the following are rational expressions versus being only fractional expressions.

$$\frac{3x^2 + 2x^{1/2} - 5}{3x + 2}$$

$$\frac{4x^5 + 2x^3 - \sqrt{5}}{5x^3 + 2}$$

Operations on Rational Expressions

- Since rational expressions are a subset of fractional expressions, all the rules for operating on fractions can be applied to rational expressions.
- Let a, b, c, and d be real numbers, variables representing real numbers, or polynomial expressions.

Operations on Rational Expressions

- Addition Rule: $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
- Multiplication Rule: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
- Division Rule: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
- Cancellation Rule: $\frac{ac}{bc} = \frac{a}{b}$

Skills

- Apply the operation rules to examples of your choice.
- HINT: Often it is helpful to factor the numerator and denominator, cancel like terms if appropriate, and then apply the rule.

Polynomial Division Pattern

- Polynomial division follows a pattern that is similar to long division of integers. So we begin by recalling the integer division algorithm via an example.

- Example: $4\overline{)137}$

Integer Division Algorithm

- First we determine how many times the **divisor** 4 goes into the 13, the first part of the **dividend**. We write this number above the division symbol, multiply it by the divisor, and subtract.

$$\begin{array}{r} 3 \\ 4\overline{)137} \\ -\underline{12} \\ 1 \end{array}$$

Integer Division Algorithm

- Now repeat this process until the divisor is less than the remaining part of the dividend.

$$
\begin{array}{r}
34 \\
4\,\overline{)\,137} \\
-\underline{12} \\
17 \\
-\underline{16} \\
1
\end{array}
$$

Division Algorithm for Integers

- We can write this result as
  - Dividend = Divisor · Quotient + Remainder
  - $137 = 4 \cdot 34 + 1$
- Where the remainder is less than the divisor

Polynomial Division Algorithm

- We can modify the long division algorithm for integers and use it for polynomial division.

- Example: $x-3\overline{\big)3x^3-5x+2}$

Polynomial Division Method

$$
\begin{array}{r}
3x^2 + 9x + 22 \\
x - 3 \overline{\big) 3x^3 + 0x^2 - 5x + 2} \\
-(\underline{3x^3 - 9x^2}) \\
9x^2 - 5x \\
-(\underline{9x^2 - 27x}) \\
22x + 2 \\
-(\underline{22x - 66}) \\
68
\end{array}
$$

Polynomial Division Algorithm

- We can write this result as
  - Dividend = Divisor · Quotient + Remainder

$$3x^3 - 5x + 2 = (x-3) \cdot (3x^2 + 9x + 22) + 68$$

- Where the degree of the remainder is less than the degree of the divisor