



Polynomial Expressions

- Section P.3



Definition of Polynomial

- An algebraic expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

- Where all **coefficients** a_i are real numbers, $a_n \neq 0$
- The **degree** n is a nonnegative integer



Skills

- Which of the following are polynomial expressions?

$$x^4 - x^{-2} + 2x - 9$$

$$x^6 - \frac{1}{2}x^3 + \sqrt{2}x - 5$$



Operations on Polynomials

- We can combine polynomials in a number of ways
 - Addition of polynomial expressions
 - Subtraction of polynomial expressions
 - Multiplication of polynomial expressions



Addition of Polynomials

- We can add polynomials by using the commutative and associative properties of real numbers to group like terms. Add the following polynomial expressions.

$$(2x^5 + 5x^2 + 3x - 9) + (4x^2 - 7x + 2)$$

$$*SOLUTION* \quad 2x^5 + 9x^2 - 4x - 7$$



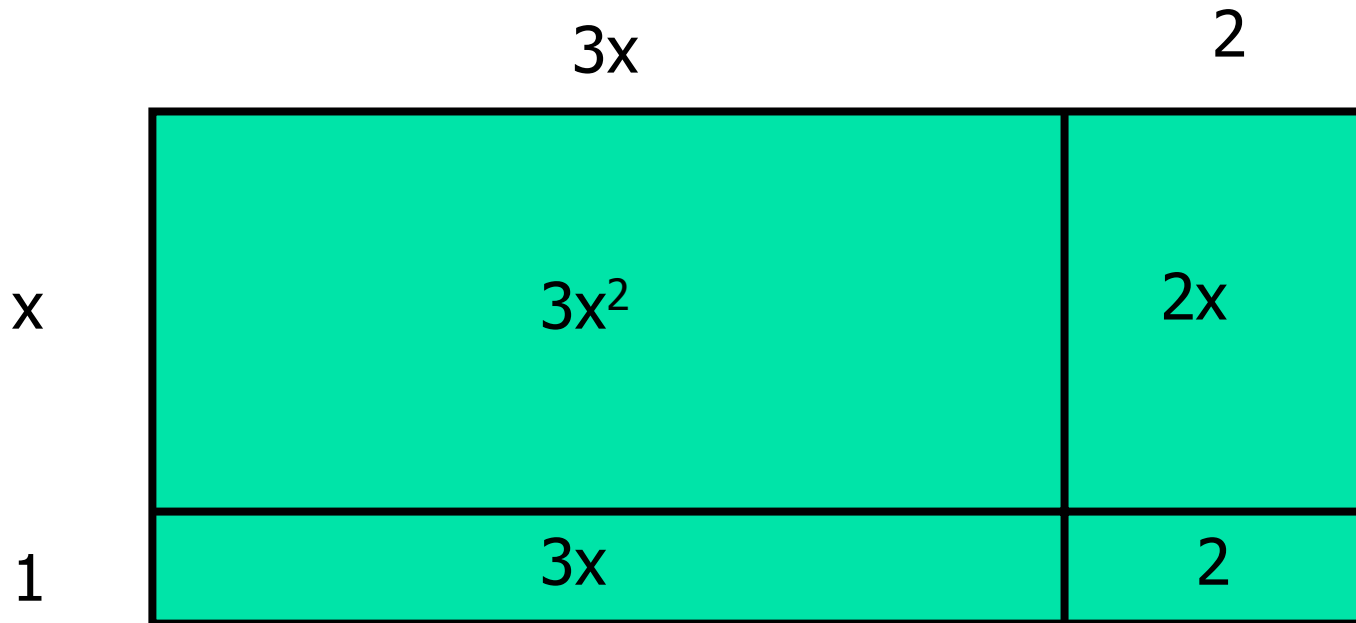
Multiplication of Polynomials

- Multiplying two polynomials requires the distributive property. The rule for multiplication can be interpreted geometrically.



Geometric Representation

- $(x + 1)(3x + 2)$



SOLUTION $3x^2 + 5x + 2$



Multiplication Rule

- Multiplying polynomials can be done by applying the distributive property
- Try this: $(5x + 2)(2x - 4)$

$$(5x + 2)(2x - 4) = 5x(2x - 4) + 2(2x - 4)$$

$$(5x + 2)(2x - 4) = (10x^2 - 20x) + (4x - 8)$$

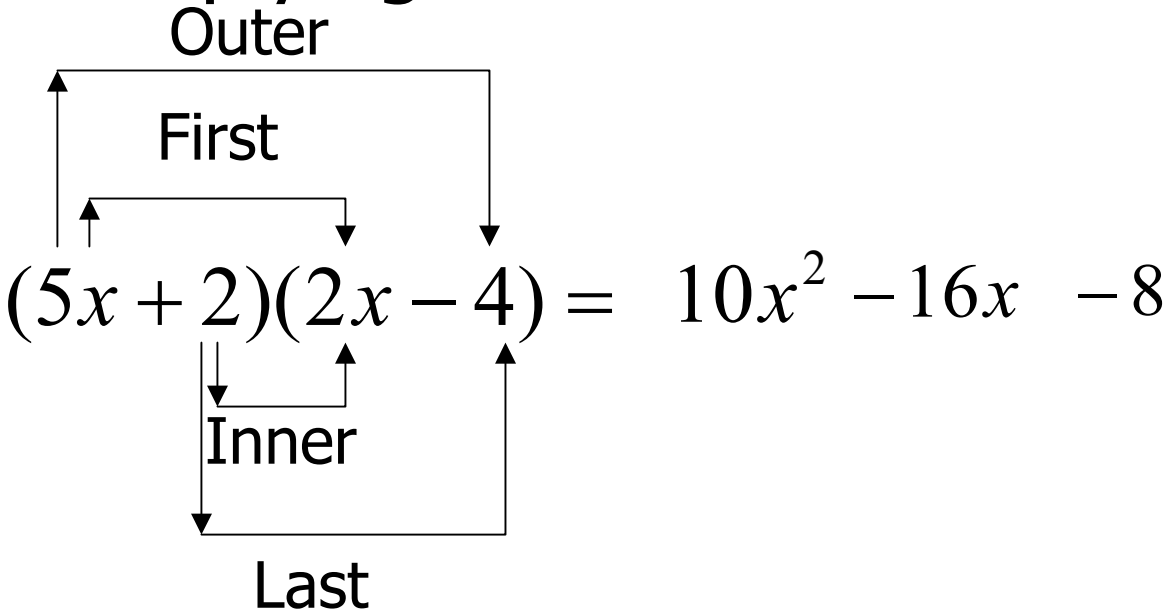
$$(5x + 2)(2x - 4) = 10x^2 + (-20x + 4x) - 8$$

$$(5x + 2)(2x - 4) = 10x^2 - 16x - 8$$



Multiplication Rule

- FOIL Mnemonic – a memory aid for multiplying two binomials.


$$(5x + 2)(2x - 4) = 10x^2 - 16x - 8$$

Outer

First

Inner

Last



Factoring Polynomials

- Relate factoring polynomials to factoring integers
- Factoring is looking for patterns and testing cases



Factoring Polynomials

- Factor $6x^2 + 7x - 20$ over the integers
- Looking for two binomial factors
- $6x^2 + 7x - 20 = (\quad)(\quad)$
- Examine the signs. How can we tell what the signs will be?
- Pattern: Second sign is $-$ so the signs are opposite. Why?

$$6x^2 + 7x - 20 = (\quad + \quad)(\quad - \quad)$$

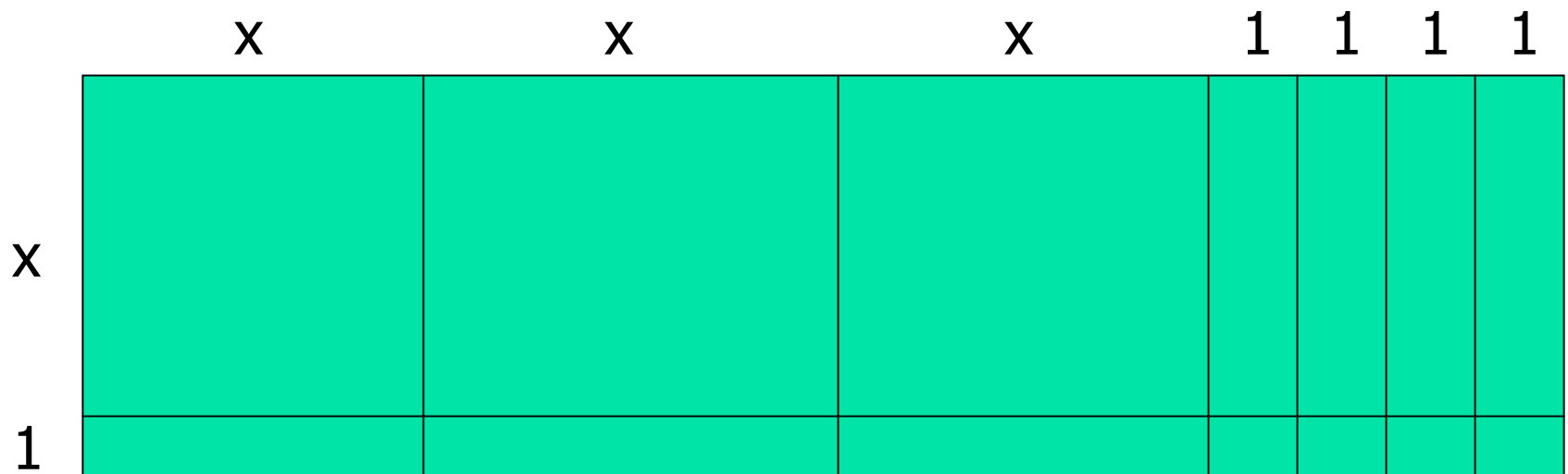


Factoring Polynomials

- What are the possible factors of the first term and last term?
- Solution: $6x^2$ factors are $x \cdot 6x$, $2x \cdot 3x$
- Solution: 20 factors are $1 \cdot 20$, $2 \cdot 10$, $4 \cdot 5$
- Guess and check – need difference of $7x$ for middle term
- $6x^2 + 7x - 20 = (2x + 5)(3x - 4)$

Geometric Representation

- If a polynomial factors over the integers, regions representing x^2 , x and units can be arranged in a rectangle.



$$(x + 1)(3x + 4)$$



Special Factor Rule

- Factor the following to determine some special factoring rule

$$x^2 - y^2$$