Exponents

- Section P.2

Exponential Notation for Positive Integers

- Let b be a real number or a variable representing a real number. Let n be a positive integer.
- $b^n = b \cdot b \cdot b \cdot b \cdots b$ where there are n b's

- Example: $8^5 = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$
- Example: $x^4 = x \cdot x \cdot x \cdot x$

Product Rule for Exponents

- What happens to the exponents when we multiply $5^3 \cdot 5^{5?}$

- Solution:
- $5^3 \cdot 5^5 = (5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5)$
- $5^3 \cdot 5^5 = 5^{3+5} = 5^8$

Exponential Rules

- We want the Product Rule for Exponents to hold for powers that are not positive integers as well. Determine the definition of each of the following exponential rules by ensuring that the Product Rule for Exponents holds.

Zero Exponent

- For any real number a, what does a^0=?

- Solution
- We want to define a zero power so that $a^m \cdot a^0 = a^{m+0} = a^m$
- What must a^0 equal for this to happen?
- $a^0 = 1$

Negative Exponent

- For any real number a, what does a^{-m} = ____?

- Solution
- We want to define a negative power so that $a^m \cdot a^{-m} = a^{m+(-m)} = a^0 = 1$
- What must a^{-m} equal for this to happen?
- $a^{-m} = 1/a^m$ so a negative power means reciprocal

Division Rule for Exponents

- For any non-zero real number a, what does a^m/a^n = ____?
- Solution
- Using the rule for negative exponents $a^m/a^n = a^m \cdot a^{-n} = a^{m-n}$
- So when we divide expressions with the same base, we subtract powers.

Other Exponential Rules

- Determine the rules for the following by expanding the expression.
- $(ab)^n$ = ______
- $(ab)^n = a^n \cdot b^n$
- $(a/b)^n$ = ______
- $(a/b)^n = a^n / b^n$
- $(a^m)^n$ = ______
- $(a^m)^n = a^{m \cdot n}$

Skills

- Use the Exponential Rules to simplify the following expressions to a common form having
  - Only positive exponents
  - All like terms combined
  - Constant portion reduced to lowest terms

Skills Practice

- $(3x^2)(2x^3)$
- $(24x^5)/(8x^{-2})$
- $(5x^5/y^2)^{-2}$

Rational Exponents

- How do we extend the notion of exponents to the rational numbers? What is $a^{1/n}$?
- Solution:
- Using the Power to a power rule, examine $a^{n/n} = (a^n)^{1/n} = a$. So the power 1/n undoes the power n. What operation undoes taking an nth power?
- Example: a^2 can be undone by taking $\sqrt{a}$ that is $\sqrt{a^2} = a$. So we define

$$a^{1/n} = \sqrt[n]{a}$$

Radical Rules

- Simplifying expressions that have radicals can be done by converting the radical to a rational power and then applying the exponential rules. Try one of these three examples.

$$\sqrt{9x^4} \qquad \sqrt[3]{54x^4} \qquad \sqrt{12xy}\sqrt{3xy^4}$$

Rationalizing the Denominator

- Common simplified form for radical expressions is
  - All factors removed from radical
  - Index of radical reduced to lowest terms
  - Rationalize the denominator (no radical left in the denominator)

Skills Practice

- Simplify the following radical expression

$$\frac{\sqrt{25x^3}}{\sqrt{x^4}}$$