Linearization

- Section 4.5

- Linearization
  - Process of transforming curvilinear data to make it linear
  - Allows for ease in determining trends and selecting the most appropriate model

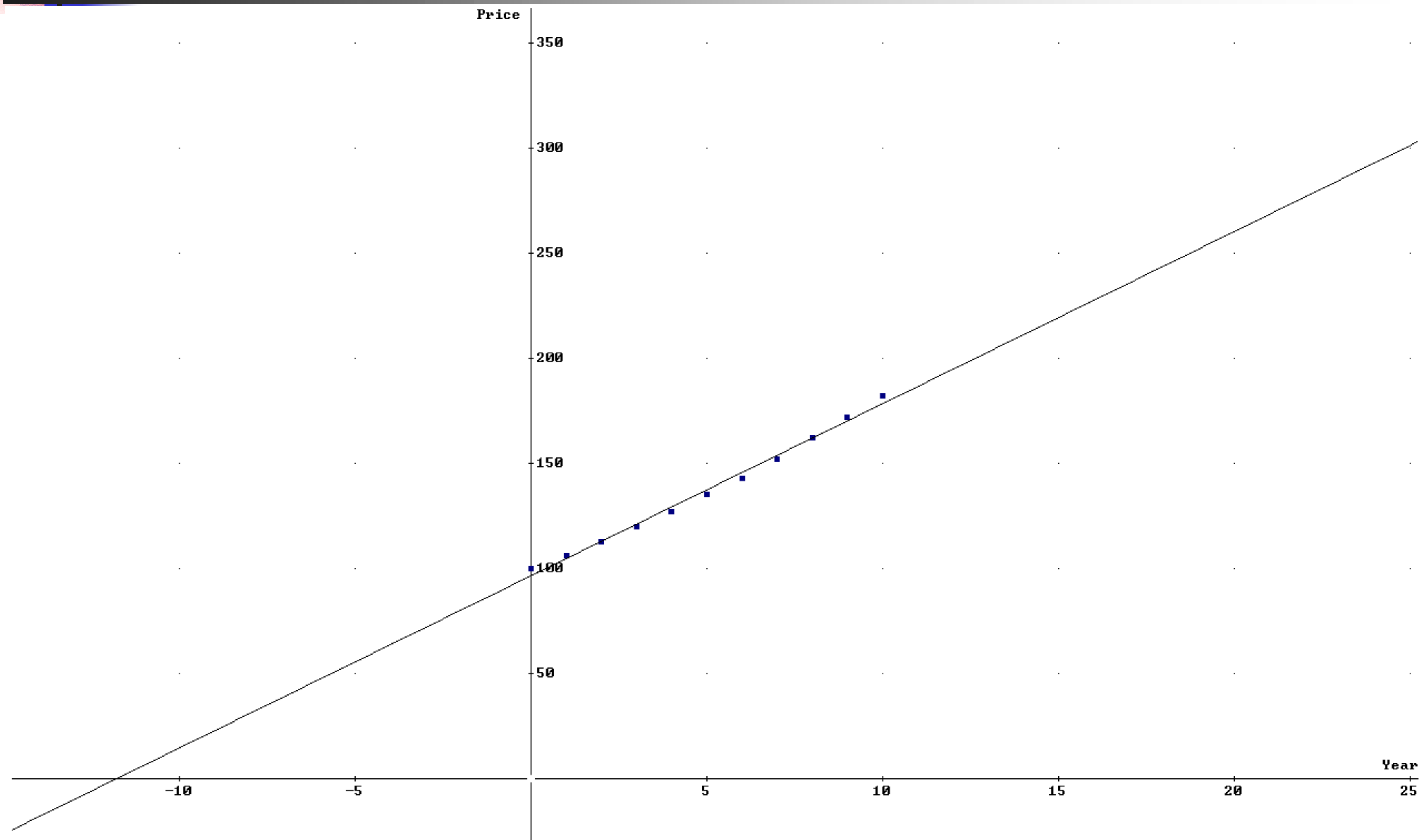
Model:
Consumer Price Index

The CPI allows comparisons of the costs of goods and services across years. If the base year for $100 of goods and services is 1967, determine the cost of these goods in 2002.

- Plot the data and determine if it is linear or curvilinear. Let 1967 be year t = 0.

Year	Price
1967	100
1968	106
1969	113
1970	120
1971	127
1972	135
1973	143
1974	152
1975	162
1976	172
1977	182

CPI Data
Scatter Plot and Best Fit Line



CPI Data is Curvilinear

- Is the CPI data modeled by a power function or an exponential function?

$$f(x) = a \cdot x^m \qquad f(x) = a \cdot e^{k \cdot x}$$

- Linearization of the data allows us to answer this question.

Linearizing Exponential Data

- We use the inverse logarithmic function to linearize exponential data. Suppose our exponential model is $y = a \cdot e^{k \cdot x}$

$$\ln y = \ln(a \cdot e^{k \cdot x})$$

$$\ln y = \ln a + \ln e^{k \cdot x}$$

$$\ln y = \ln a + k \cdot x$$

$$Let\ Y = \ln y,\ B = \ln a,\ then\ linear\ in\ \ln y$$

$$Y = B + k \cdot x$$

Linearizing Exponential Data

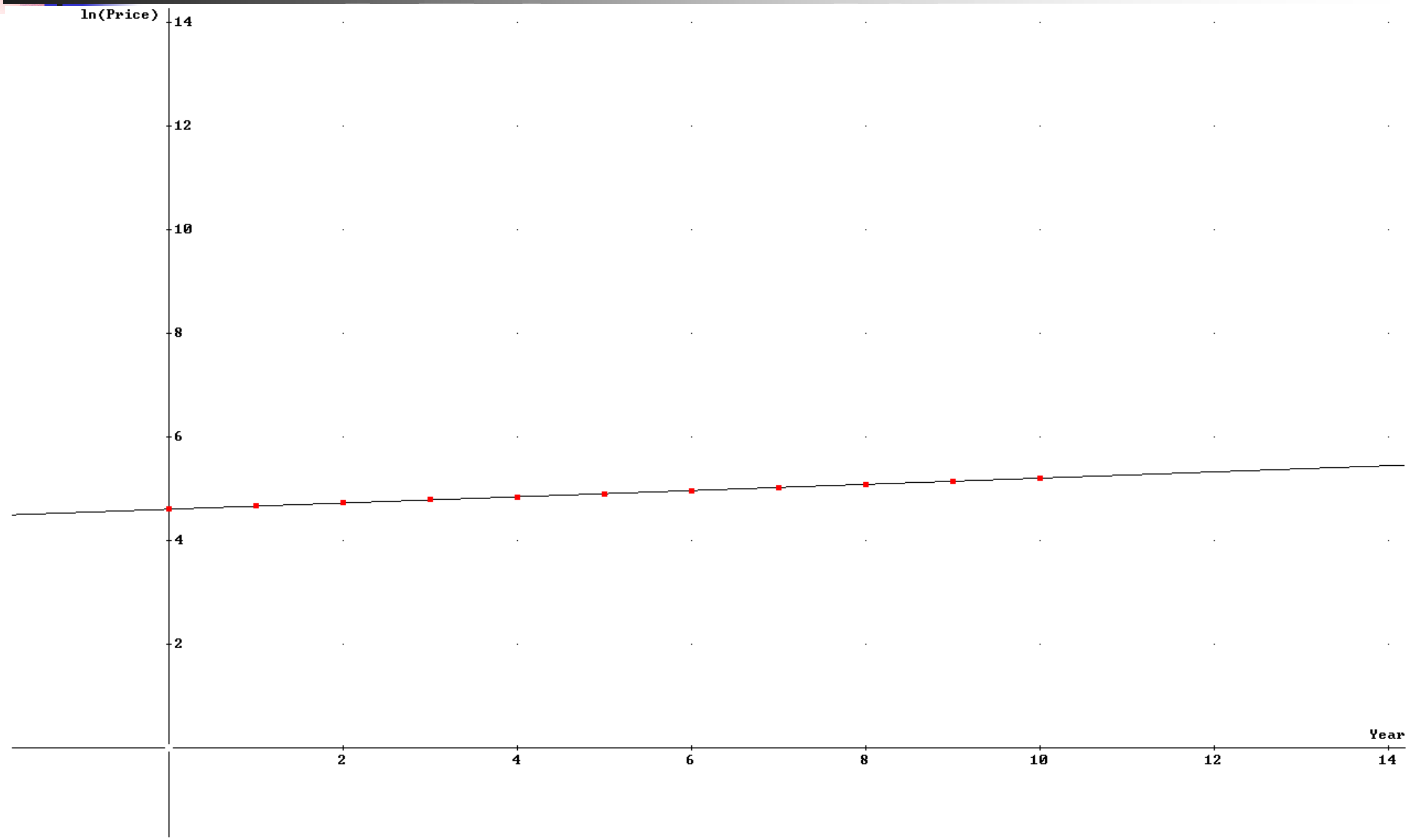
- If the model is exponential, then transforming data (x, y) by taking the Semi-log (x, ln y) will linearize the data.
- Fit a line to the linearized data.
- Reverse the above procedure to find the exponential model.

Linearizing CPI data

- Natural logarithm of the dependent variable Price results in a nearly constant rate of change of k = 0.06
- Indicates the original data has an exponential model

Year	Price	ln(Price)
1967	100	4.605
1968	106	4.663
1969	113	4.727
1970	120	4.787
1971	127	4.844
1972	135	4.905
1973	143	4.963
1974	152	5.024
1975	162	5.088
1976	172	5.147
1977	182	5.204

Linearization CPI Data: (t, ln p) data and Best Fit Line



CPI Exponential Model

- Best Fit Line for Semi-log data (t, ln p)

$$P = 0.06t + 4.61$$

$$\ln(p) = 0.06t + 4.61$$

$$p = e^{0.06t} \cdot e^{4.61}$$

$$p(t) \approx 100.48e^{0.06t}$$

CPI Exponential Model

- Determine CPI in 2002
  - t = 2002 – 1967 so t = 35

$$p(35) = 100.48e^{0.06 \cdot 35} \approx \$820.54$$

Model: Territory

- A wild animal maintains a territory which they defend from animals of the same species.
  - How is the weight of the animal related to its territorial area?
  - Not linear, test to see if power function model.

Weight	Area
10	20
20	50.5
50	168
100	417
150	709

Linearizing Power Data

- We use the inverse logarithmic function to linearize power data. Suppose our power model is $y = a \cdot x^m$

$$\ln y = \ln(a \cdot x^m)$$

$$\ln y = \ln a + \ln x^m$$

$$\ln y = \ln a + m \cdot \ln x$$

$$Let\ Y = \ln y,\ X = \ln x,\ B = \ln a,$$

$$then\ linear\ in\ \ln x\ and\ \ln y : Y = B + k \cdot x$$

Linearizing Power Data

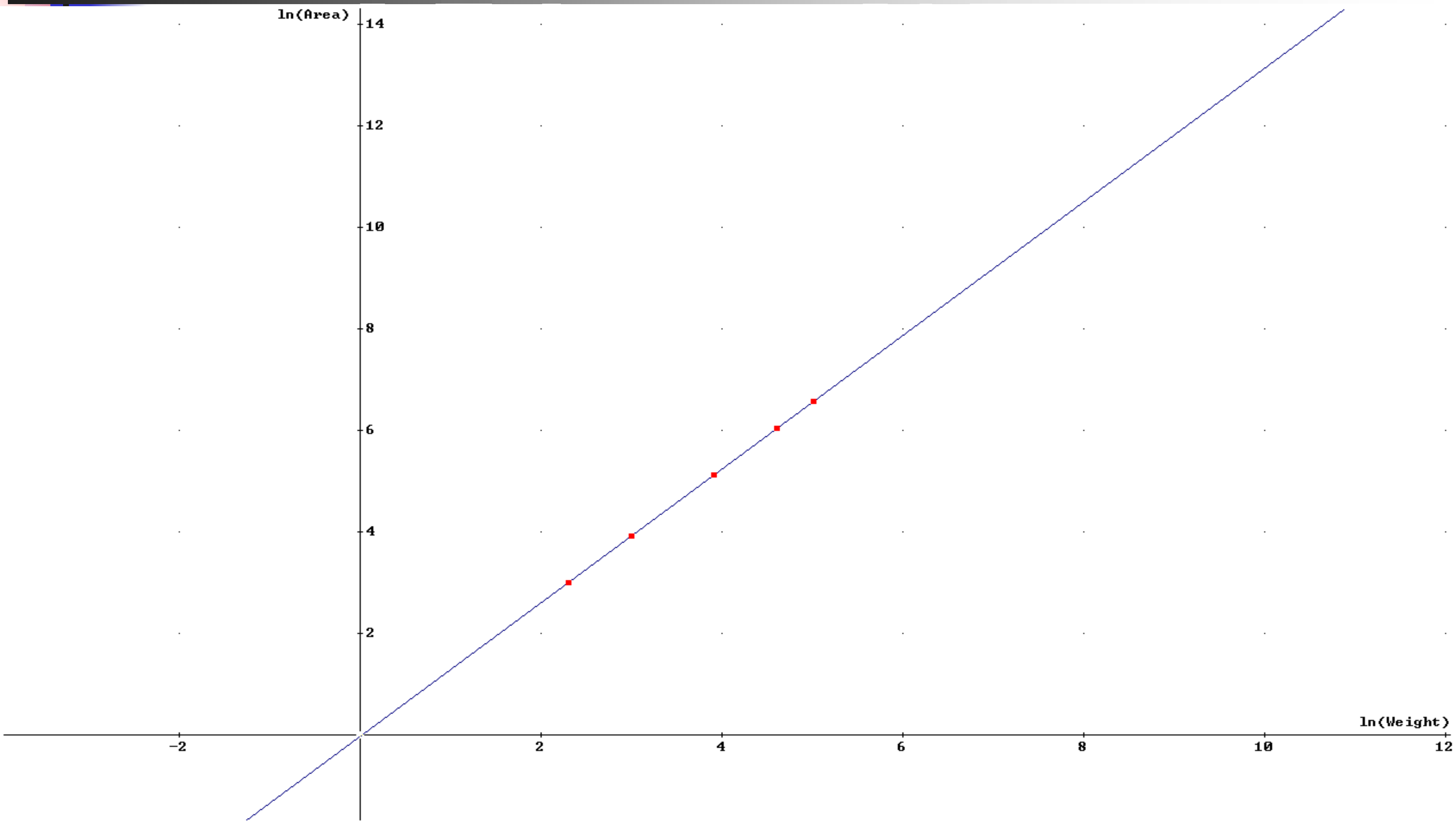
- If the model is power function, then transforming data (x, y) by taking the Log-log (ln x, ln y) will linearize the data.
- Fit a line to the linearized data.
- Reverse the above procedure to find the power model.

Linearizing Territory data

- Natural logarithm of both variables results in a linear trend
- Indicates the original data has a power model

Weight	Area	Ln(Weight)	Ln(Area)
10	20	2.303	2.996
20	50.5	2.996	3.923
50	168	3.912	5.124
100	417	4.605	6.033
150	709	5.010	6.564

Territory data Log-log Plot and Line of Best Fit



Territory Power Model

- Best Fit Line for Log-log data (ln w, ln t)

$$T = 1.32 \cdot W - 0.03$$

$$\ln(t) = 1.32\ln(w) - 0.03$$

$$e^{\ln(t)} = e^{1.32\ln(w) - 0.03}$$

$$t = e^{\ln(w)^{1.32}} \cdot e^{-0.03}$$

$$t(w) = 0.97 \cdot w^{1.32}$$

Territory Power Model

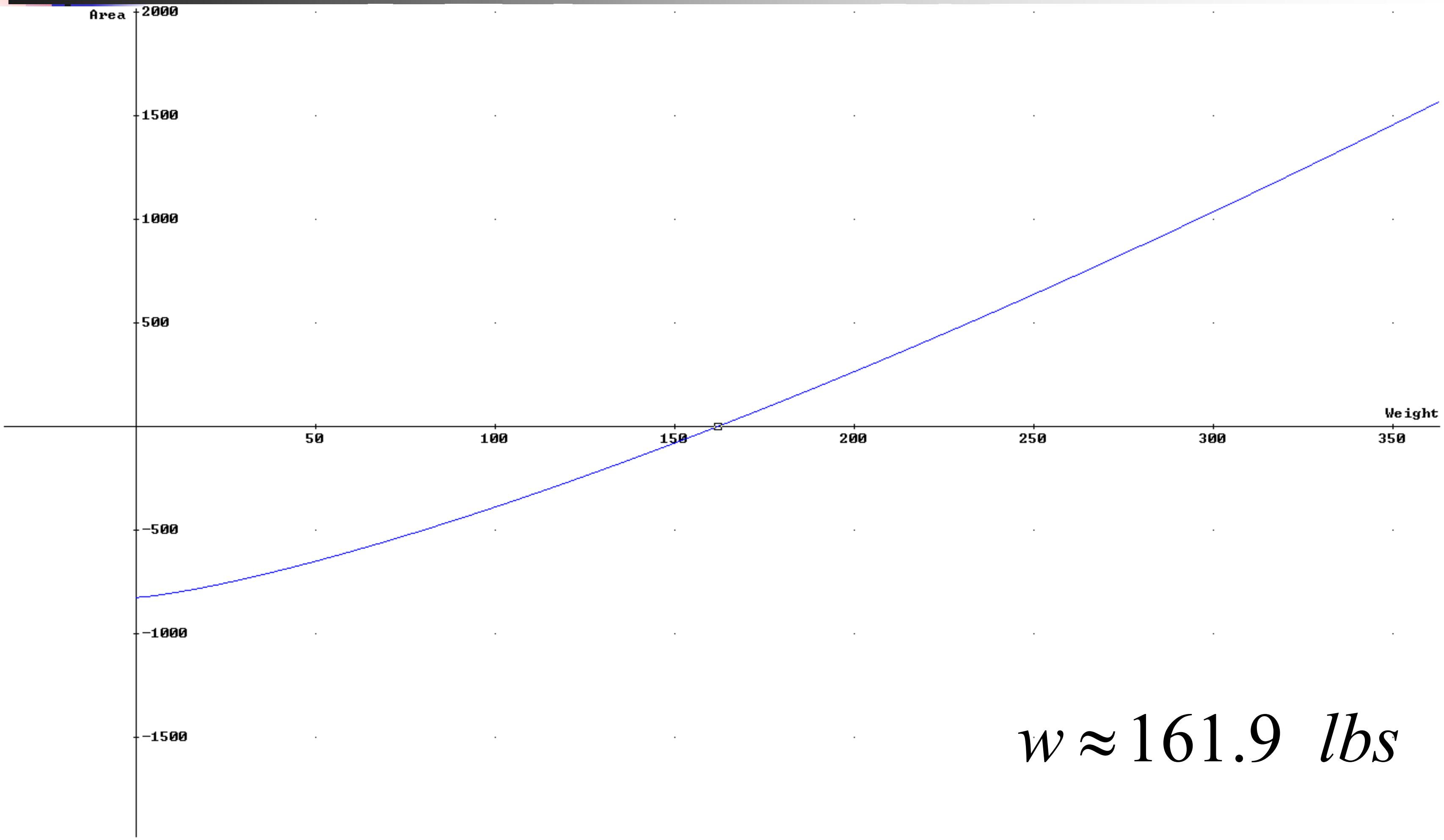
- Determine when the territorial area is 800 acres.

$$t(w) = 0.97 \cdot w^{1.32}$$

$$800 = 0.97 \cdot w^{1.32}$$

$$w^{1.32} = 824.74$$

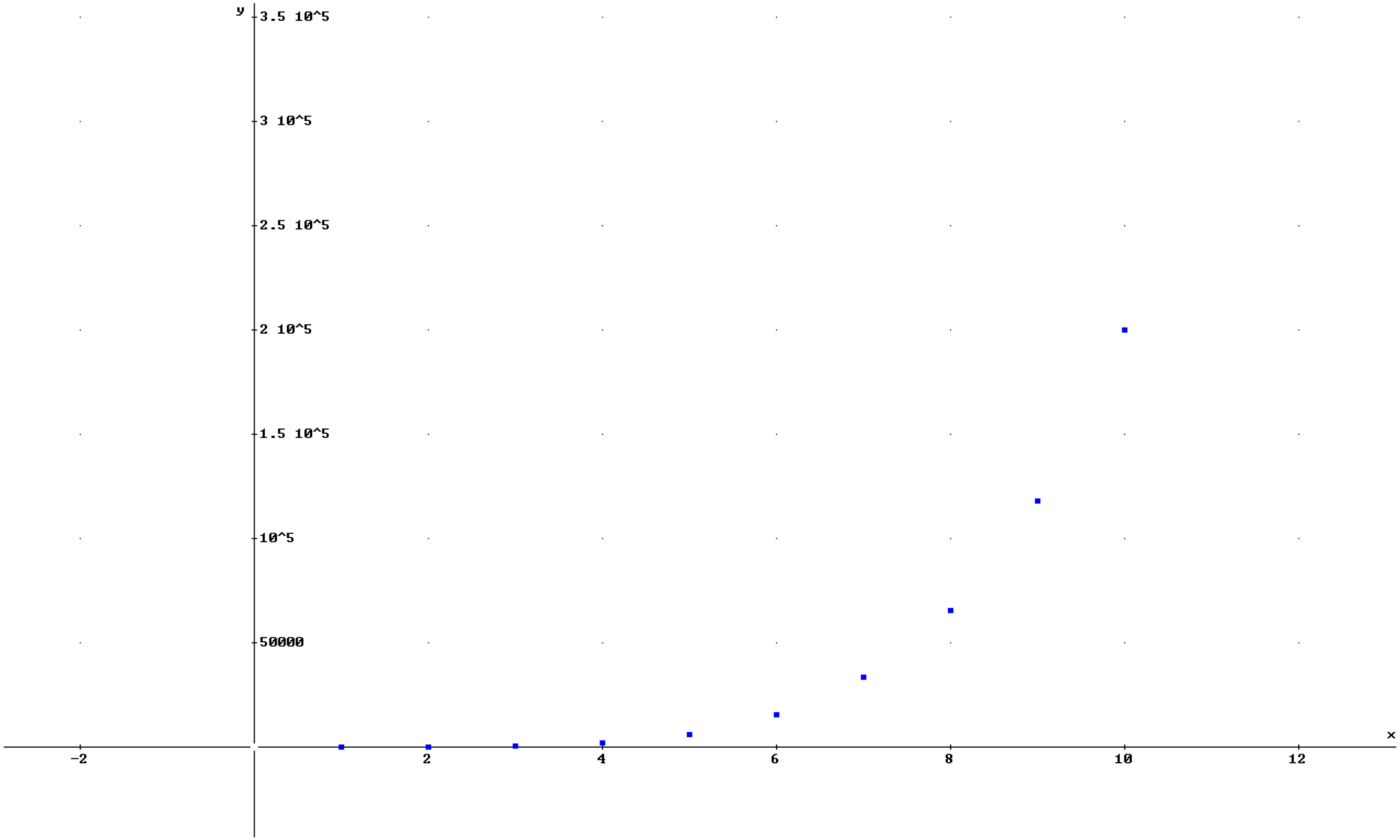
Graphical Solution: $y = w^{1.32} - 824.74$



$w \approx 161.9 \;\; lbs$

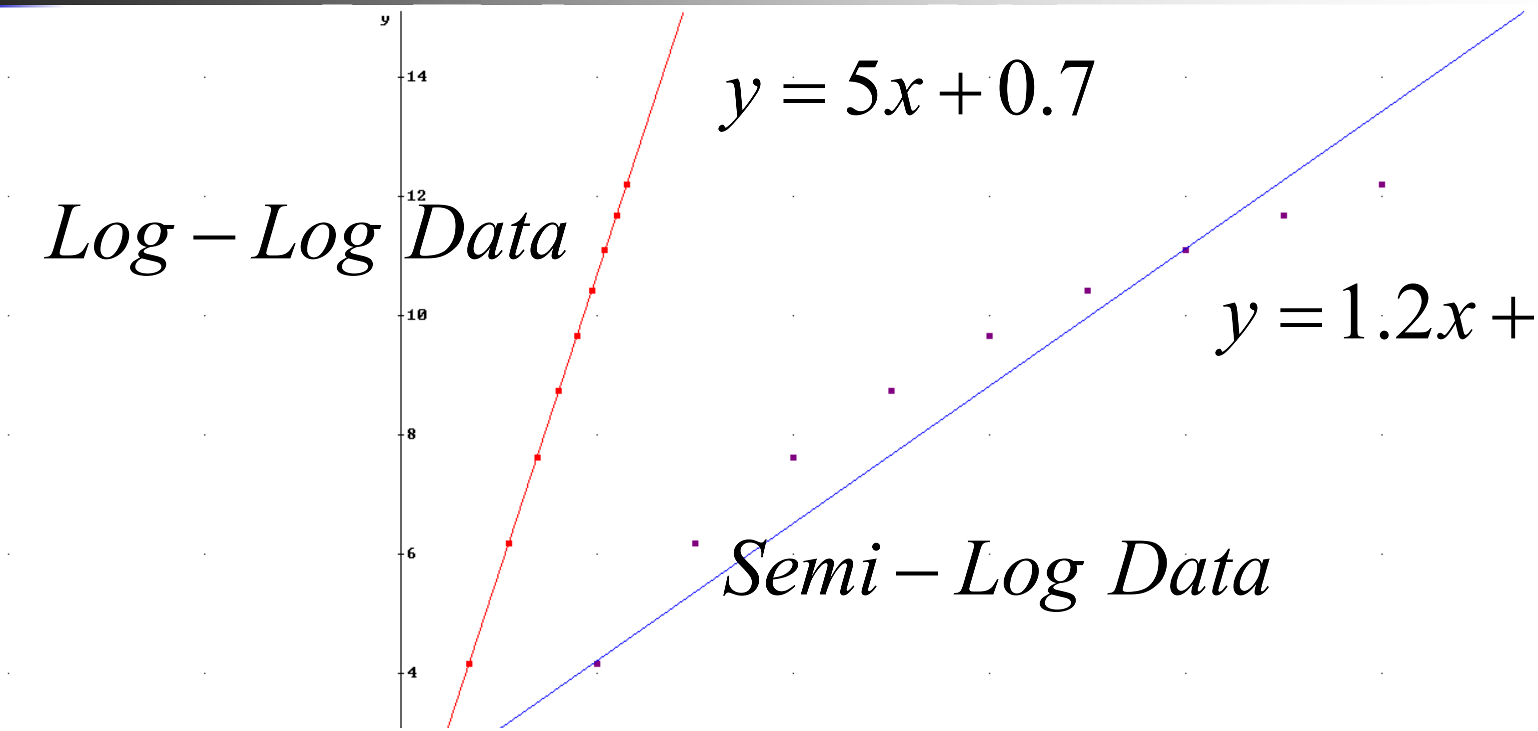
Classroom Participation

Find the model for data using linearization



Classroom Participation

Find the model for data using linearization



- Use linearization to determine a model for the original data? Is it a power function or an exponential function?

$$y = 2x^5$$