Exponential and Logarithmic Equations

- Section 4.4

- Variety of methods of solving exponential equations
  - Similar bases
  - Logarithm with same base
  - Common logs and Natural logs

Model: Carbon Dating

- The radioactive element carbon-14 is measured by a Geiger Counter which counts disintegrations per minute (dpm). At the time of death the dpm is 15.3. Find the age of a chair leg measuring 10.14 dpm if the model is

$$d(t) = d(0) \cdot e^{-0.00012t}$$

- Solve $10.14 = 15.3 \cdot e^{-0.00012t}$

Algebraic Solution: Common Base

- Exponential function is 1-1 so

$$If\ b^x = b^y\ then\ x = y$$

- Thus if bases are the same, then we can set the powers equal
  - Example: Solve $16^{2x} = 8^{4x-4}$
  - Participation Activity: Solve $3^{5x} = 9^{2x-1}$

- Can we solve the Carbon Dating Problem using this method? $10.14 = 15.3 \cdot e^{-0.00012t}$

Algebraic Solution: Inverse Logarithmic Function

- Two logarithmic bases that most technology will work with
  - Common Logarithm – base 10
  - Natural Logarithm – base e
- If base of exponential equation is 10 or e, apply the appropriate logarithmic inverse function

Algebraic Solution

- Solve the Carbon Dating equation

$$10.14 = 15.3 \cdot e^{-0.00012t}$$

  - Isolate the exponential term
  - Use the logarithm to bring down the exponent
  - Solve the resulting equation

- Will this method work if the base of the exponent is not 10 or e?

Model: Electric Energy

- The demand for electric energy in the U.S. was 800 billion kilowatt hours in 1965. Increase in demand is predicted to be 8% per year. When will the demand be 8000?
  - Model $d(t) = 800 \cdot (1 + 0.08)^t$
  - Equation $8000 = 800 \cdot (1.08)^t$

Model: Electric Energy

- Solve using natural logarithm

$$8000 = 800 \cdot (1.08)^t$$

$$10 = (1.08)^t$$

$$\ln 10 = \ln(1.08)^t$$

- Can we bring down the power if the base of the logarithm is not the same as the base of the exponent?

Logarithmic Power Property

$$\log_b c^x = x \cdot \log_b c$$

$$Let\ u = \log_b c\ then\ b^u = c$$

$$Raise\ to\ x\ \ then\ b^{ux} = c^x$$

$$Take\ \log_b\ \ then\ \log_b b^{ux} = \log_b c^x$$

$$Now\ \log_b b^{ux} = ux\ and\ u = \log_b c$$

$$So\ \log_b c^x = x \cdot \log_b c$$

Model: Electric Energy

- We can pull down the power even when the base is not the same

$$\ln 10 = \ln(1.08)^t$$

$$\ln 10 = t \cdot \ln(1.08)$$

$$t = \frac{\ln 10}{\ln 1.08} \approx 29.92$$

  - Participation Activity: Solve $9^x = 15$
  - Solution: $x = \log(15)/\log(9) \approx 1.23$

Logarithmic Model

- Humans tend to forget what they have learned. A psychological experiment determined the percent of material remembered r after m months is

$$r(m) = 75 - 6 \cdot \ln(m+1)$$

  - Determine how many months pass before 60% of the material is forgotten.
  - Equation: $60 = 75 - 6 \cdot \ln(m+1)$

Logarithmic Equation

- Solve $60 = 75 - 6 \cdot \ln(m+1)$
  - Isolate the logarithmic term
  - Apply the inverse exponential function to remove the logarithm
  - Solution: $m = e^{5/2} - 1 \approx 11.18$
  - Participation Activity: Solve $6 + \log(x-5) = 8$
  - Solution: $x = 105$

Logarithmic Properties

- Solve: $\log_3(2x-3)+\log_3(x+2)=2$
- Verify the Logarithmic Sum Property

$$\log_b c+\log_b d=\log_b(c\cdot d)$$

- Verify the Logarithmic Difference Property

$$\log_b c-\log_b d=\log_b \frac{c}{d}$$

Logarithmic Equation

- Solving logarithmic equations with two logarithmic terms requires using the sum or difference Logarithmic Properties
  - Example: $\log_3(2x-3)+\log_3(x+2)=2$
  - Solution: $x = \frac{5}{2} \; or \; x = -3$
  - Extraneous solutions are possible, so check them
  - Participation Activity: Solve

$$\log_5(3x-2)-\log_5 x = 1$$

Solving Mixed Equations

- Exponential or logarithmic equations that mix bases

$$3^x + 5^x = 12$$

- Equations that mix exponential and logarithmic expressions

$$3^x + \ln x = 5$$

- Equations that mix transcendental and algebraic expressions

$$3^x + 5x^2 = 5$$

Solving Mixed Equations

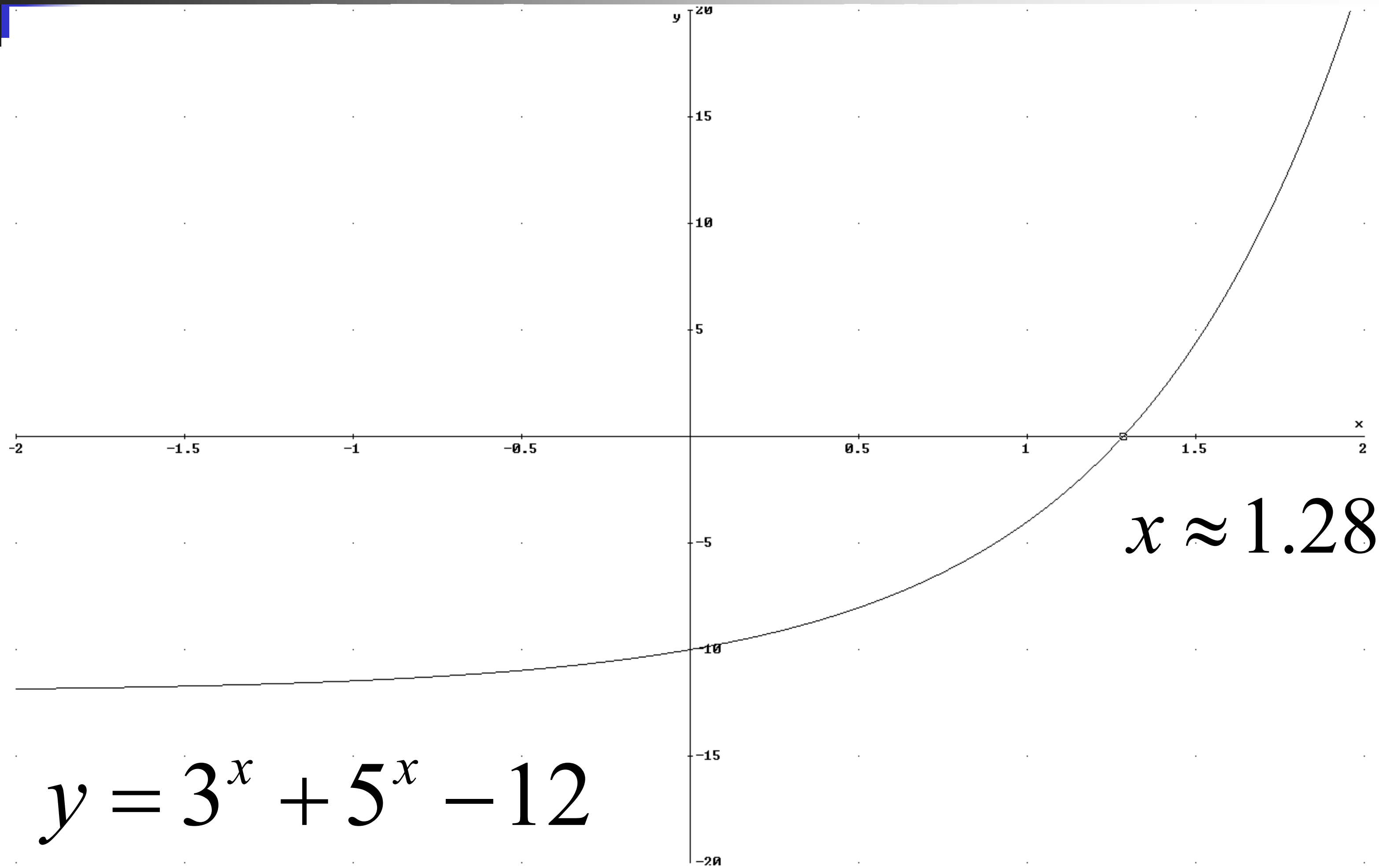
- Use Graphic Method to solve these mixed equations

$$3^x + 5^x = 12$$

$$3^x + \ln x = 5$$

$$3^x + 5x^2 = 5$$

$3^x + 5^x = 12$



$x \approx 1.28$

$y = 3^x + 5^x - 12$