



Logarithmic Functions

- Section 4.3
- Logarithm comes from the Greek *logos*, meaning “reckoning, ratio” and *arithmos* meaning “number”
- John Napier used logs to reckon large values in astronomy – Napier’s Bones were the predecessor of the calculator



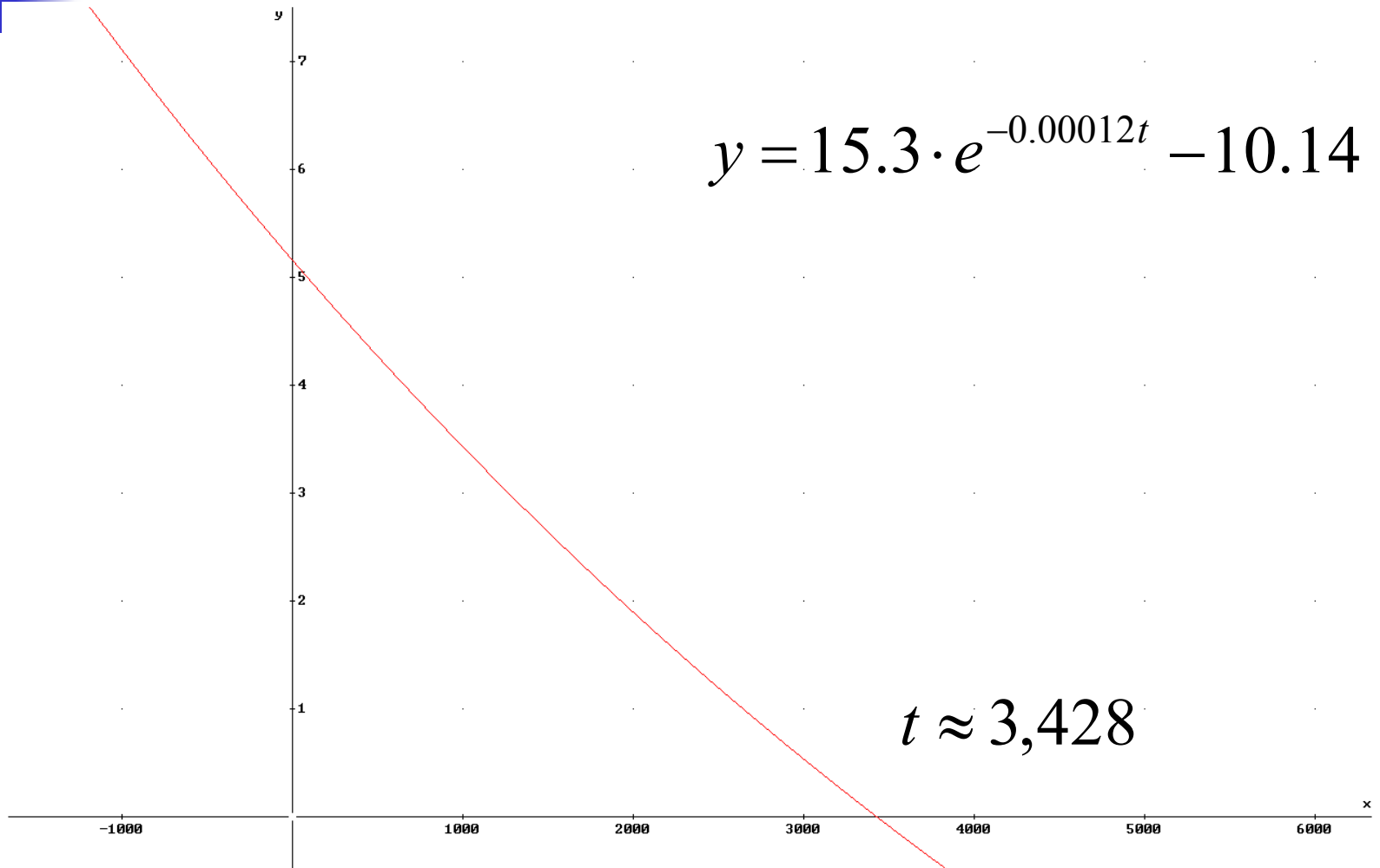
Model: Carbon Dating

- The radioactive element carbon-14 is measured by a Geiger Counter which counts disintegrations per minute (dpm). At the time of death the dpm is 15.3. Find the age of a chair leg measuring 10.14 dpm if the model is

$$d(t) = d(0) \cdot e^{-0.00012t}$$

- Solve $10.14 = 15.3 \cdot e^{-0.00012t}$

Graphic Solution

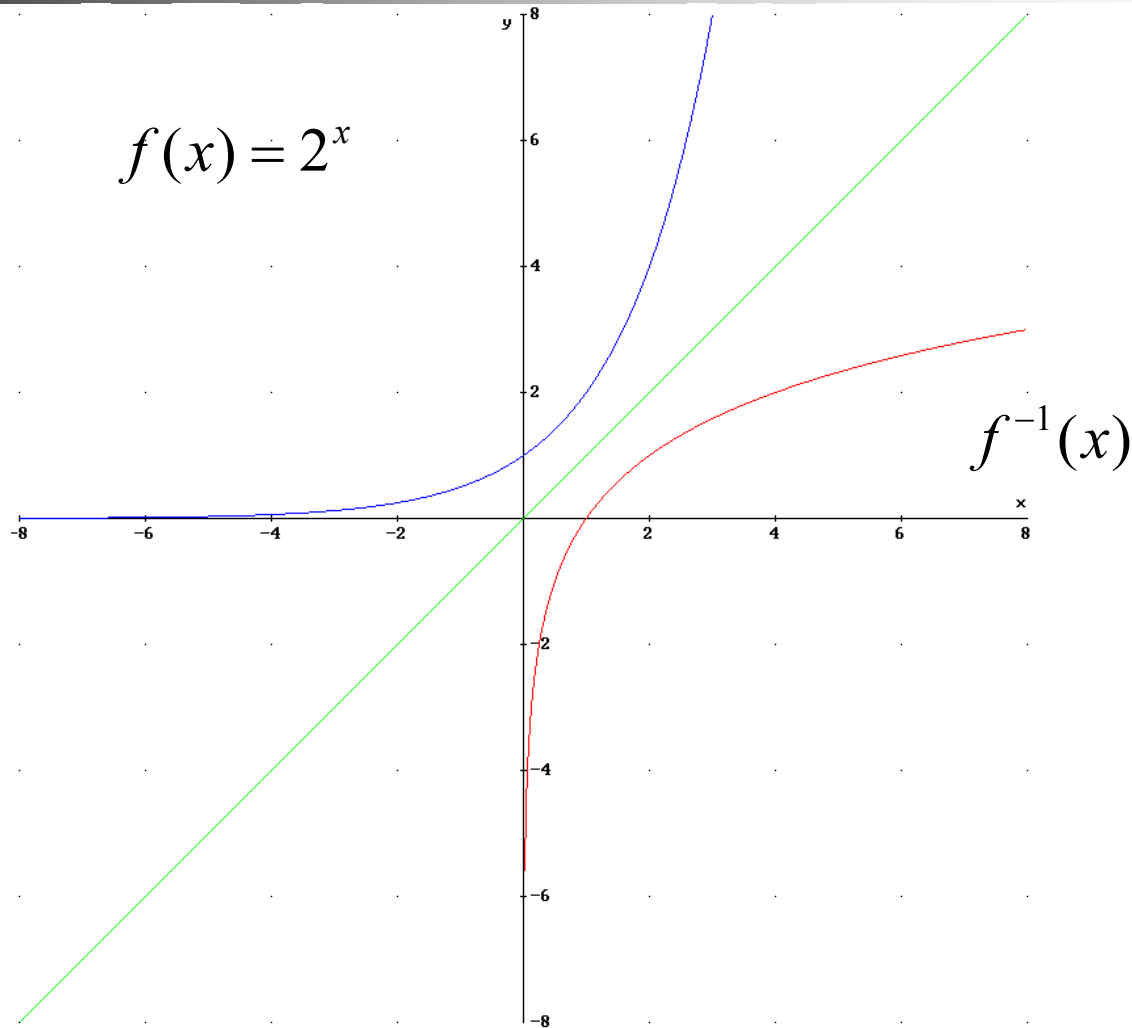




Algebraic Solution

- Need an inverse function for the exponential function
 - Does an inverse exponential function exist?
 - Solution: Yes, it is 1-1 so inverse exists
 - Let $f(x) = 2^x$. Sketch a graph of the inverse function and identify the characteristics of the function

Exponential Function Inverse





Characteristics of Inverse

- Domain: $(0, +\infty)$
- Range: $(-\infty, +\infty)$
- X-intercept: $(1, 0)$
- Increasing/decreasing? *Increases*
- Concavity? *Down*

- End Behavior:

$$x \rightarrow 0, f^{-1}(x) \rightarrow -\infty$$

$$x \rightarrow +\infty, f^{-1}(x) \rightarrow +\infty$$

- Asymptote: $x = 0$

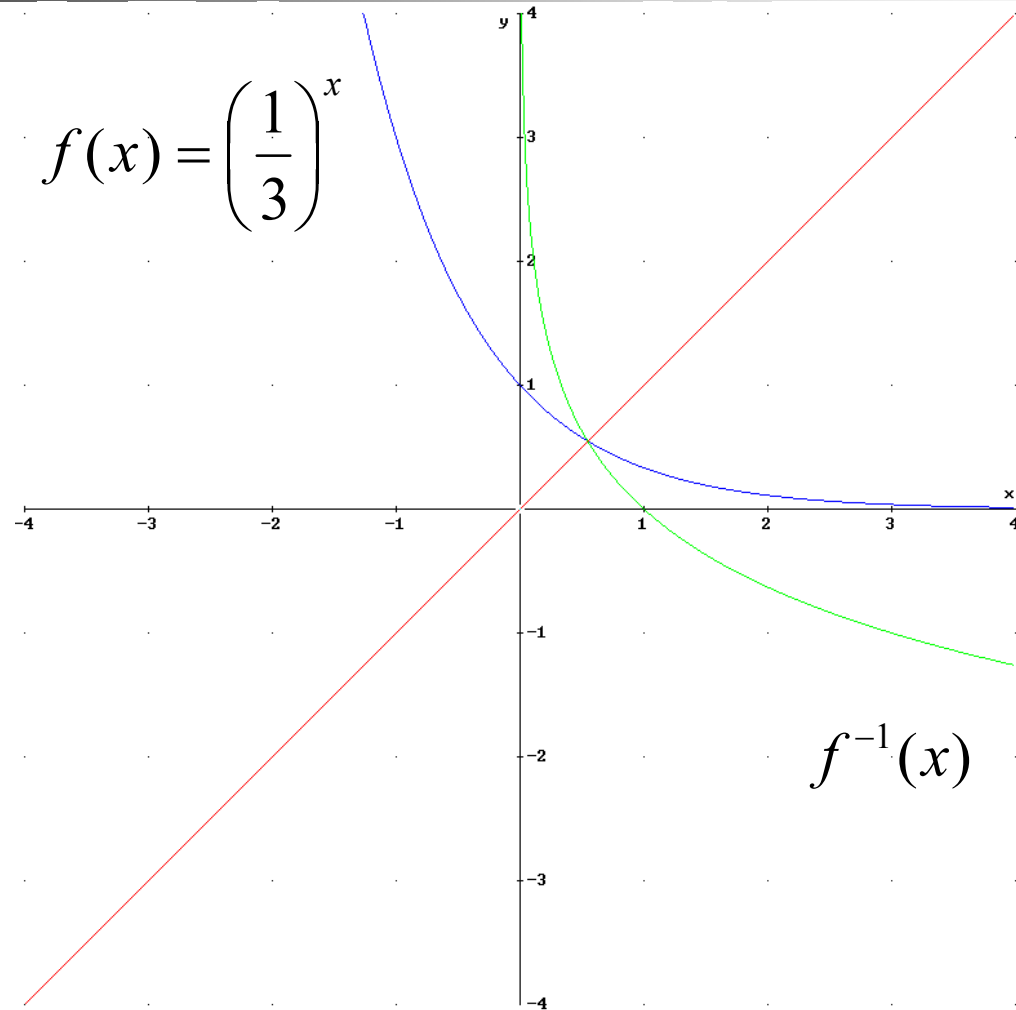


Characteristics of Inverse

- If the base of the exponential function is $0 < b < 1$, what are the inverse function characteristics?
- Sketch a graph of the inverse for $0 < b < 1$.

- Example: $f(x) = \left(\frac{1}{3}\right)^x$

Inverse Exponential Function





Logarithmic Function

- For $x > 0$ and $b > 0$, b not 1, the inverse function for the exponential function is

$$f(x) = \log_b x \Leftrightarrow b^y = x$$



Logarithmic Properties

- Use the definition of logarithm to convert between logarithmic and exponential form
- Determine value of:

$$y = \log_b 1$$

$$y = \log_b b$$

$$y = \log_b b^x$$

$$y = b^{\log_b x}$$

$$\log_b u = \log_b v, \text{ then } u = ?$$



Algebraic Solution

- Solve $10.14 = 15.3 \cdot e^{-0.00012t}$
- Isolate the exponential term
- Use the logarithm to bring down the exponent
- Solve the resulting equation



Algebraic Solution

- Participation Exercise: Solve the following exponential equations

$$3e^{3x} - 5 = 10$$

$$10^{5x+4} = 0.1$$



Logarithmic Models

- Useful for compressing scales when an event occurs over an incredible range of values
- Human range of hearing
- Earthquake intensity
- Acidity as measured by pH



Earthquake Intensity

- The Richter scale is a logarithmic scale for the magnitude m of an earthquake. The Chile quake in 1985 released $E = 1.26 \times 10^{16}$ joules of energy. Determine the magnitude of this quake.

$$m(E) = \frac{2}{3} \log_{10} \frac{E}{E_0}$$

where $E_0 = 1 \times 10^{4.4}$ is a reference quake

- Solution: $m \approx 7.8$



Earthquake Intensity

- How many times more energy was released by the Chile quake then by a reference quake?
 - Solution: Take E/E_0
 - 501,615,034,800 times as much energy