Exponential Functions: Known Relationship

- Section 4.2
- Natural base – an irrational number that occurs often in nature as a base for exponential functions
- Represented by *e* to honor Euler who discovered the number

$$e \approx 2.718......$$

Depreciation

- A computer costs $3000 and depreciates 20% per year. What is the computer worth in 5 years?
  - Why is the model an exponential function?
  - What is the constant rate of change k?
  - What is the initial value?

Exponential Decay

- Exponential decay is based on a constant rate of decrease k from an initial amount at an initial time t.

Exponential Model – Known Relationship

- Basic exponential model is

$$f(t) = a \cdot b^t$$

- How do this *a* and *b* relate to the given problem? Let's explore.

Exponential Model – Known Relationship

- Let t = 0, then how does the value of a relate to the depreciation problem?

$$f(t) = a \cdot b^t$$

$$f(0) = a \cdot b^0$$

$$f(0) = a \cdot 1 = a$$

Depreciation

What is *b* in $f(t) = a \cdot b^t$?

- A computer costs $3000 and depreciates 20% per year. What is the computer worth in 5 years?
  - Use recursive function to find closed form.

$A(0) = 3000$

$A(1) = A(0) – 0.2\,A(0) = A(0)\,(1 – 0.2)$

$A(2) = A(1) – 0.2\,A(1) = A(1)\,(1 – 0.2)$

$\quad = A(0)\,(1 – 0.2)^2$

$A(3) = ?$ $\qquad\qquad$ $A(t) = ?$

Exponential Model – Known Relationship

- So given the initial value f(0) and the constant rate of change k over a time interval Δt = 1 the exponential model is

$$f(t) = f(0) \cdot (1 + k)^t$$

Exponential Model – Known Relationship

- Participation Activity: A stock has an initial value of $2,000 per share. If the stock increases in value 9% per year, find a model for the stock value as a function of time in years.
  - SOLUTION:

$$f(t) = 2000 \cdot (1.09)^t$$

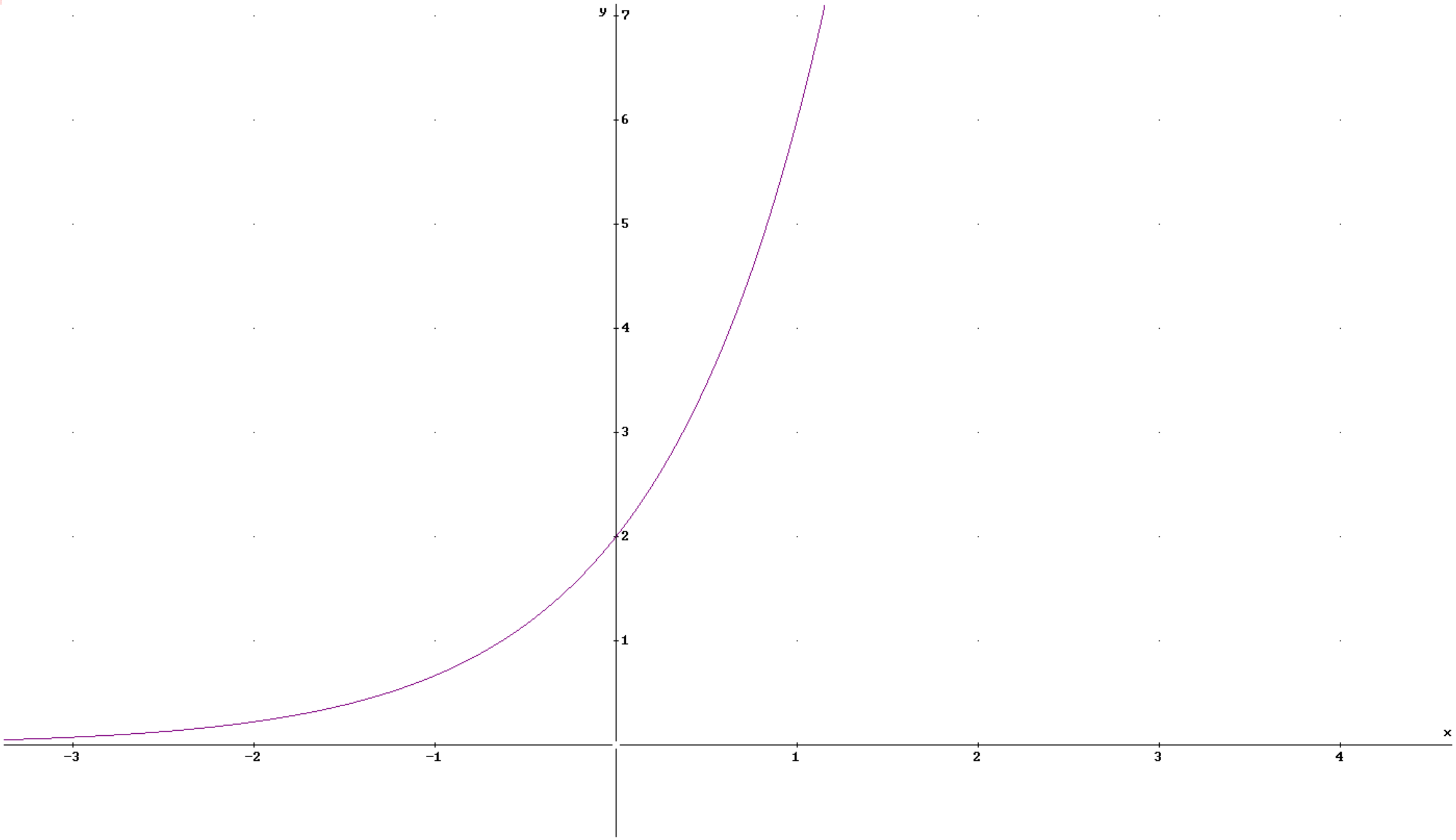
Two Basic Exponential Graphs

- Exponential Growth (U.S. Population)

$$f(x) = a \cdot b^x$$

- Where b > 1 and a > 0
  - What is the end behavior of the exponential growth function?
  - What is the intermediate behavior?
  - Is the function increasing or decreasing?
  - What is the concavity?

Exponential Growth



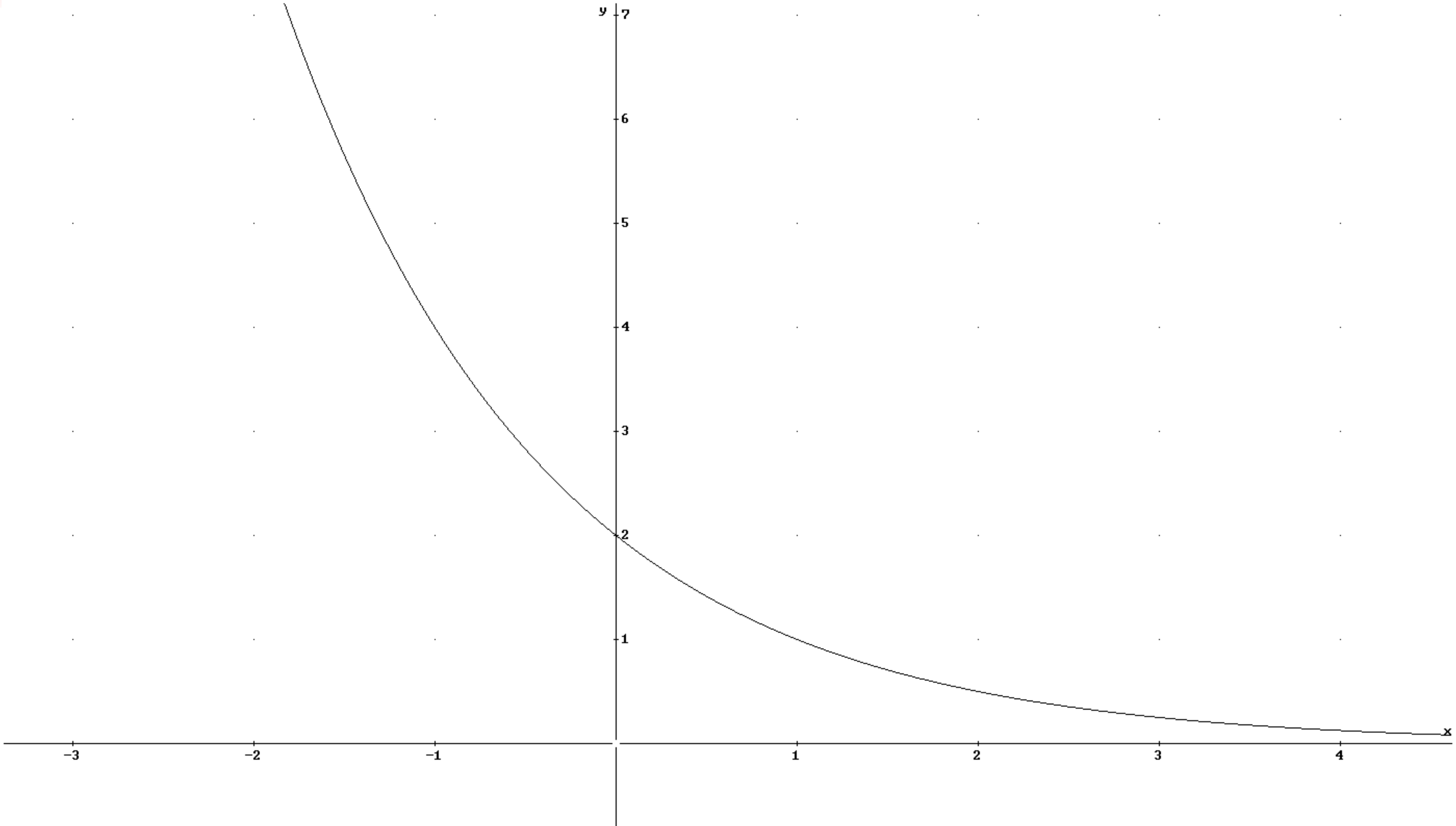
Two Basic Exponential Graphs

- Radioactive Decay (Carbon Dating)

$$f(x) = a \cdot b^x$$

- Where $0 < b < 1$ and $a > 0$
  - What is the end behavior of the exponential decay function?
  - What is the intermediate behavior?
  - Is the function increasing or decreasing?
  - What is the concavity?

Exponential Decay



Transformation of Exponential Function

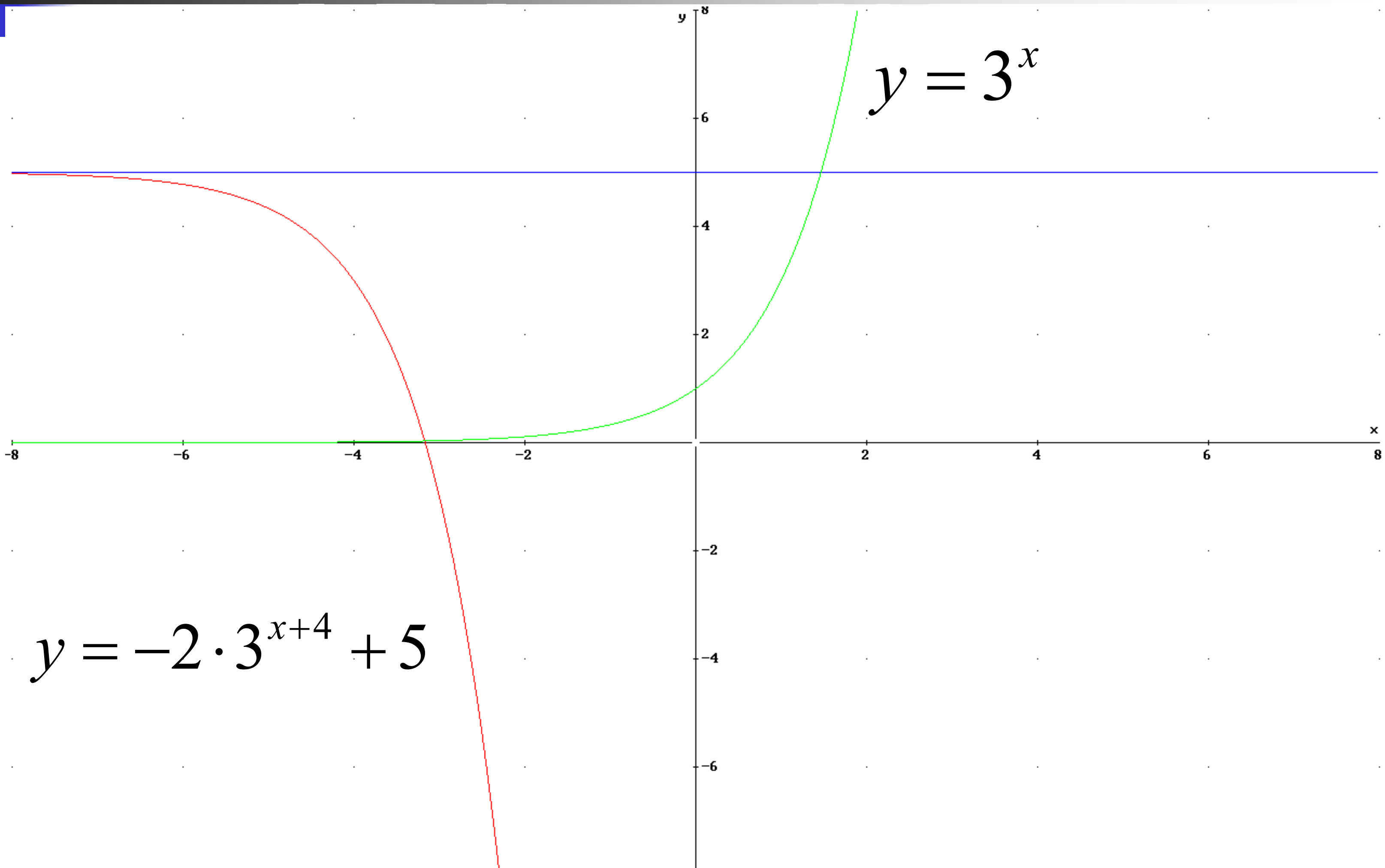
- What are the effects of a, c and d in the transformation of the exponential function

$$y = a \cdot b^{x+c} + d$$

- Example:

$$y = -2 \cdot 3^{x+4} + 5$$

Transformation: $y = -2 \cdot 3^{x+4} + 5$



Model: Carbon Dating

- The radioactive element carbon-14 has a ½ live of 5750 years. The percentage of carbon 14 present in the remains of plants or animals can be used to determine their age.

- How old is a human bone that has lost 25% of its carbon 14?

Carbon Dating

- In the carbon dating problem we are not given the initial value, only that the ½ life is 5750 years.
- When we can't determine the base *b* directly, we have to assign it a value and determine the *k* related to the base value.
- We will use *e* as the natural base value, since any base can be converted to any other base.

Carbon Dating

- Exponential model has form

$$f(t) = a \cdot e^{c \cdot t}$$

- Use ½ life to find c.

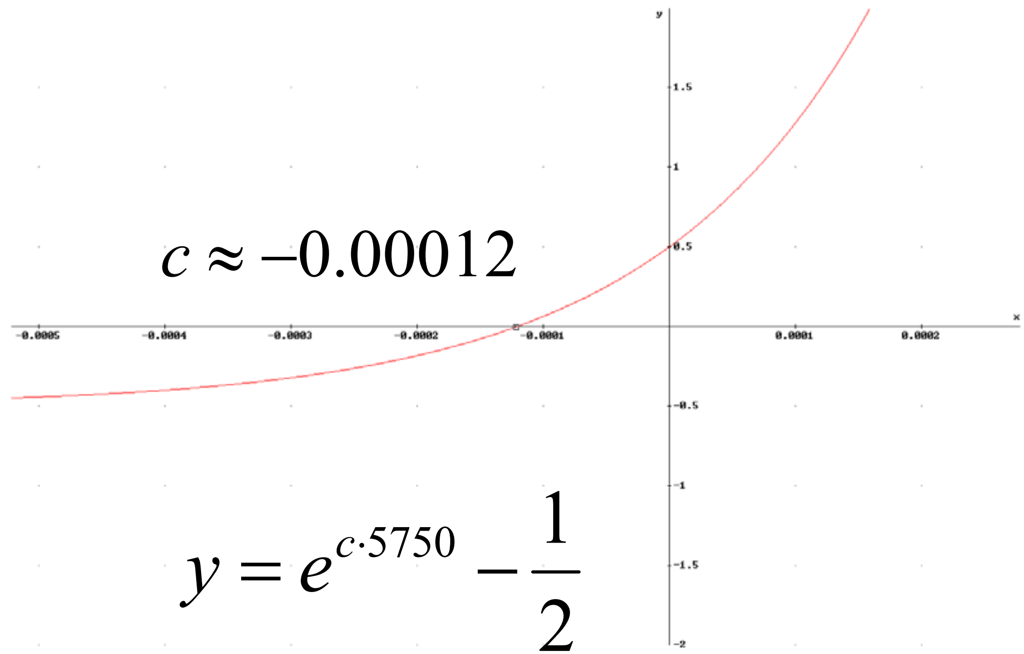
$$\frac{1}{2} f(0) = f(0) \cdot e^{c \cdot 5750}$$

$$\frac{1}{2} = e^{c \cdot 5750}$$

Carbon Dating

$$\frac{1}{2} = e^{c \cdot 5750}$$

- We do not have a means of solving this equation using algebra, so approximate c graphically.



$$c \approx -0.00012$$

$$y = e^{c \cdot 5750} - \frac{1}{2}$$

Carbon Dating

- Exponential model has form

$$f(t) = a \cdot e^{-0.00012 \cdot t}$$

- If 25% carbon-14 lost, then 75% remains so

$$0.75 f(0) = f(0) \cdot e^{-0.00012 \cdot t}$$

$$0.75 = e^{-0.00012 \cdot t}$$

Carbon Dating

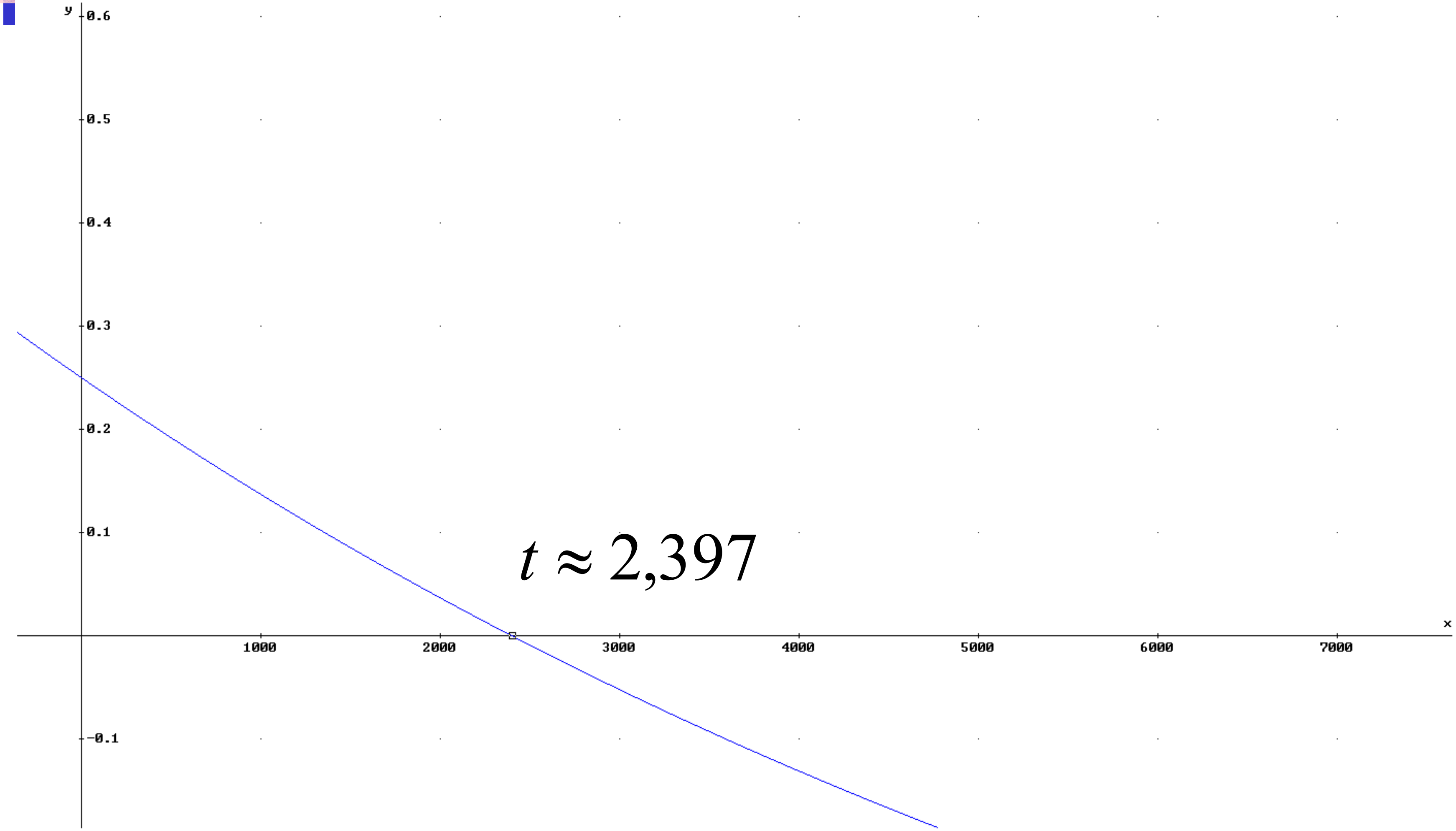
- Solve the related equation graphically

$$0.75 = e^{-0.00012 \cdot t}$$

$$y = e^{-0.00012 \cdot t} - 0.75$$

Carbon Dating – Graphic Solution

$$y = e^{-0.00012 \cdot t} - 0.75$$



$$t \approx 2{,}397$$

Carbon Dating

- The human bone is approximately 2,397 years old.
- It is clear in solving this problem that an algebraic method of solving exponential equations is needed.
- We need an inverse function for the exponential function – which is the logarithmic function.