



Rational Equations and Inequalities

- Section 3.6
- Rational Functions are useful models for
 - Inverse variation relationships
 - Proportion or ratio problems
 - Relationships approaching limits



Model: Product Exchange

- Product exchange functions give the relationship between quantities of two items that can be produced by the same factory. An oil refinery can produce gasoline, heating oil, or a combination of the two. Data was collected for gas versus heating oil production in 1000 gallon allotments
 - Determine a model for the product-exchange of gas and heating oil.

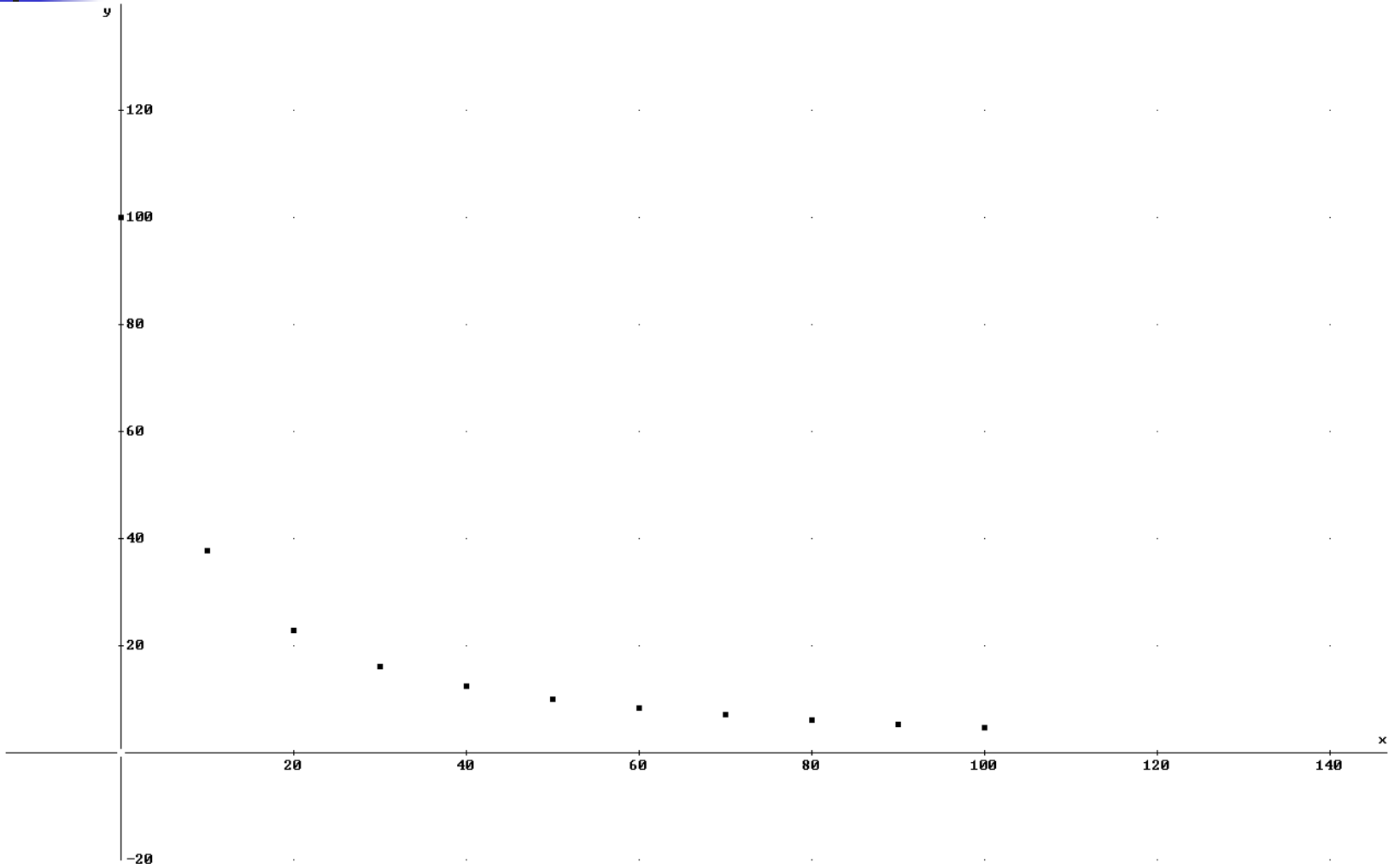
Gas	Oil
0	100
10	37.69
20	22.86
30	16.21
40	12.43
50	10
60	8.30
70	7.05
80	6.09
90	5.32
100	4.71



Model: Product Exchange

- View the graph in the next slide and address these questions
 - What functions have we studied that have the same concavity and decreasing trend as the data?
 - What in the context of the problem indicates choosing one of these functions over the other?

Product Exchange





Rational Function Model

- Serves as good model for data that approaches a fixed value or a limiting value since rational functions have asymptotes.
- Serves as a good model when the context is a ratio or comparison, such as gas versus heating oil



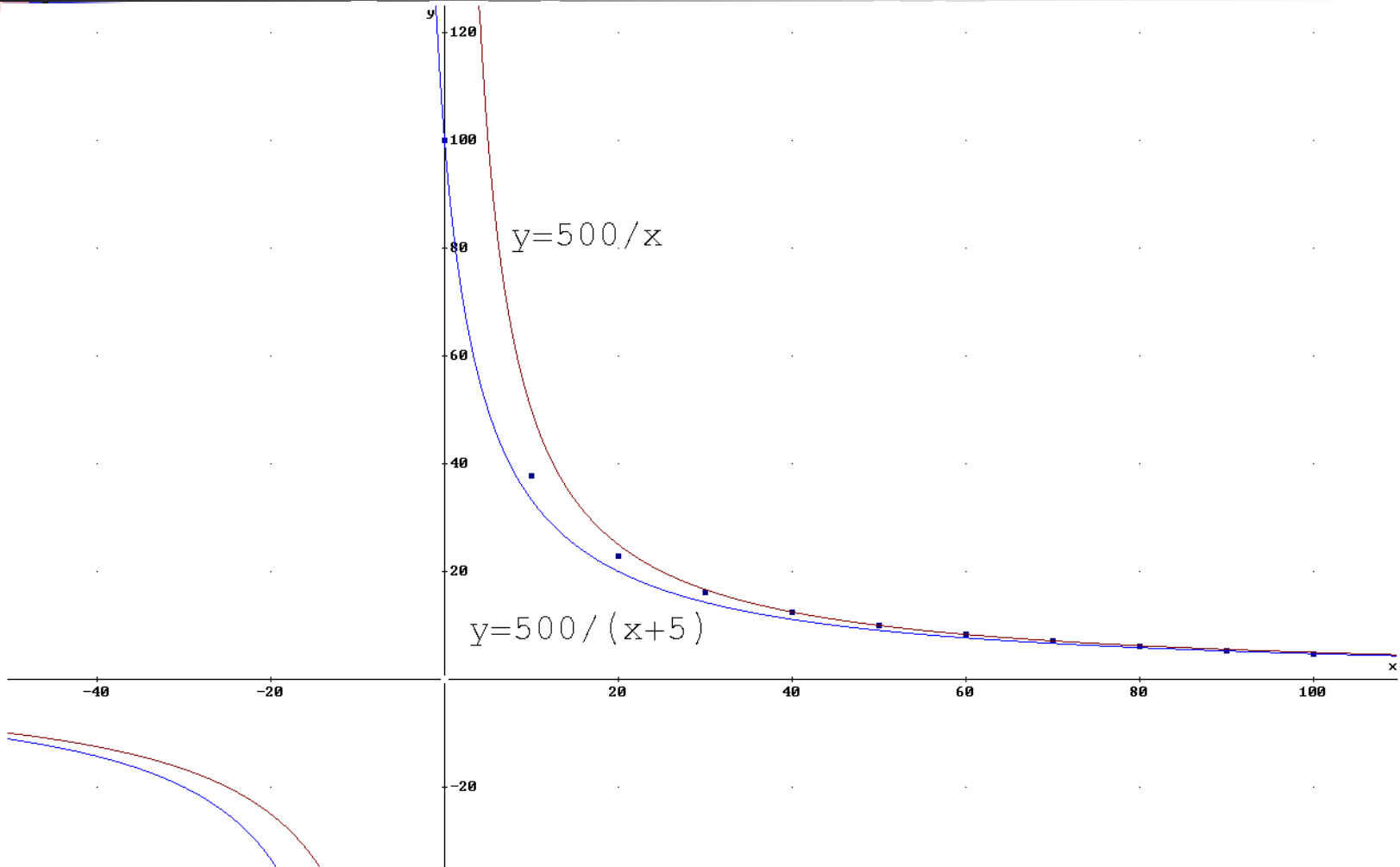
Rational Function Model

- How do we determine a rational model?
 - *Derive 5* and Graphing calculators do not fit rational functions to data
 - Estimate model using basic rational power function $y = A/x$. Will $A > 1$ or $A < 1$?
 - Solution: Using graphing technology and eyeballing the curve of best fit we get

$$g(h) = \frac{500}{h}$$

- Translate to start at $(0, 100)$ $g(h) = \frac{500}{h + 5}$

Power Function Models





Actual Model

- Difficult to find actual model
 - Linearize the data (using Exercise 51 or 53)
 - Determine from other problem context not provided

- Solution:
$$g(h) = \frac{2500 - 5h}{4h + 25}$$



Rational Equations

- Determine the heating oil production if the gas production is 15 of the 1,000 gallon allotments.

$$15 = \frac{2500 - 5h}{4h + 25}$$

- How do we solve rational equations?



Solving Rational Equations

- Convert the rational equation into a polynomial equation by multiplying by the least common denominator
- Solve the resulting polynomial equation
- Check the solutions to be sure none of them are extraneous
- Use graphic methods if analytic methods fail



Solving Rational Equations

- Solve the following example

$$\frac{1}{x-3} + 1 = \frac{6}{x^2 - 9}$$

- Class Participation Activity

$$\frac{7}{x-5} + 4 = \frac{8x}{x^2 + 4}$$



Solving Inequalities

- Solve inequalities with higher degree polynomial, radical, absolute value, or rational expressions using the graphic method

$$\frac{1}{x^3 + 5x} \geq \sqrt[3]{x^3 + 3x - 5}$$



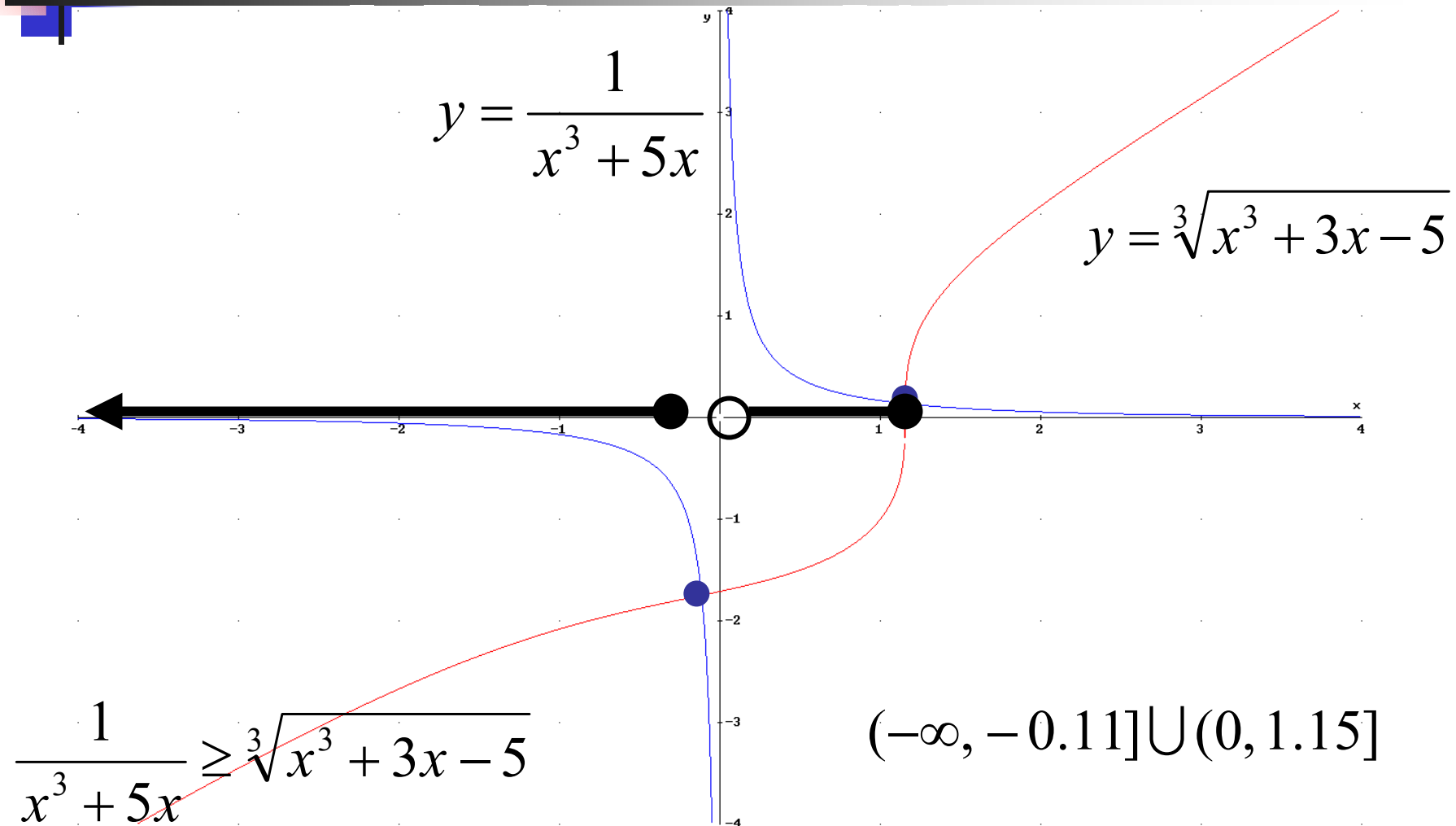
Solving Inequalities

- Either set to zero and write one related function
- Or write a related equation for each side of the inequality

$$y = \frac{1}{x^3 + 5x}$$

$$y = \sqrt[3]{x^3 + 3x - 5}$$

Solving Inequalities – Graphic Method





Transformation of Algebraic Functions

- Given any algebraic function $y = f(x)$ we can transform it.

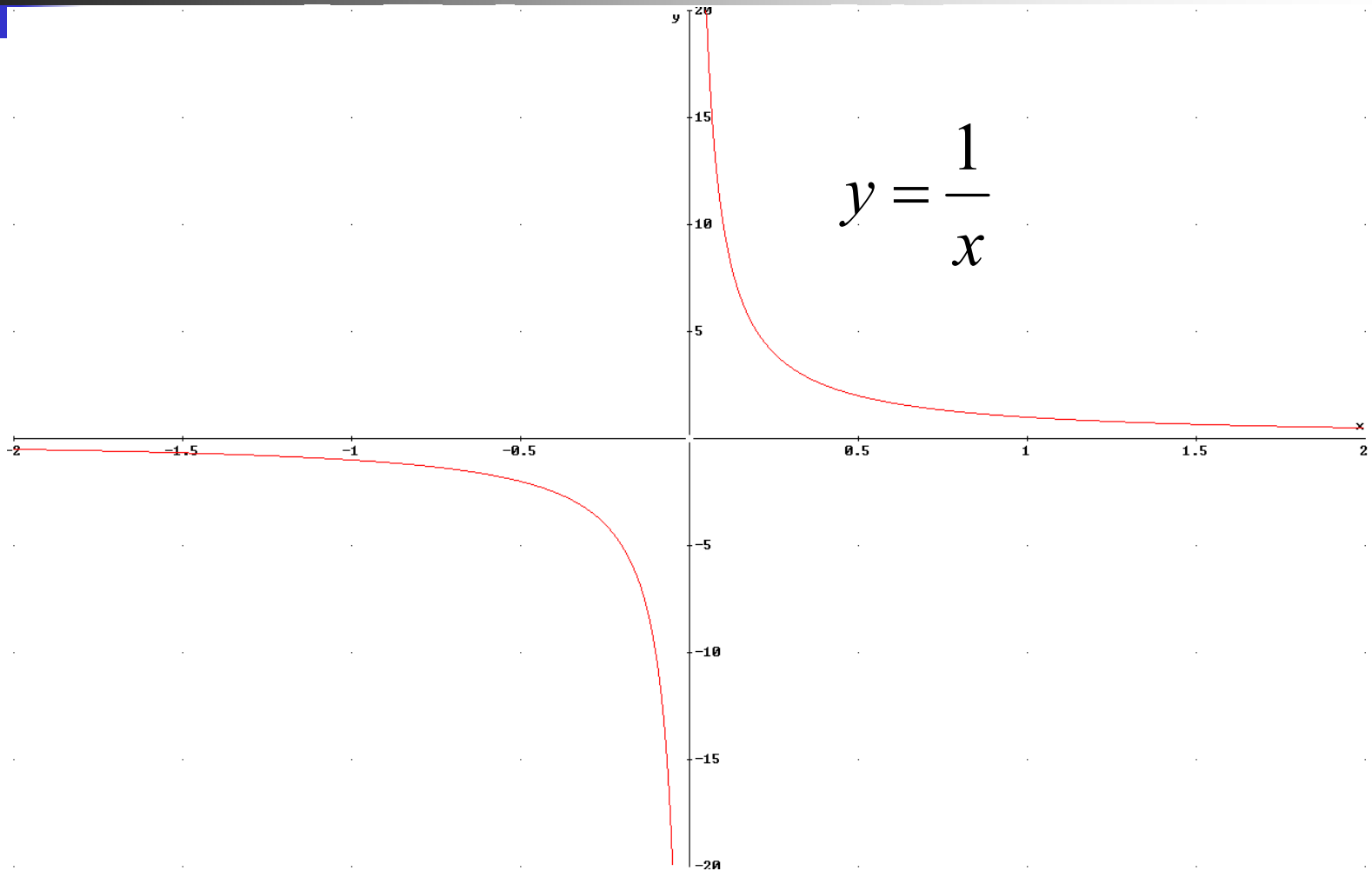
$$y = a \cdot f(x + b) + c$$

- $a > 1$ stretch, $0 < a < 1$ compress, $a < 0$ reflection in x-axis
- $b > 0$ translation left, $b < 0$ translation right
- $c > 0$ translation up, $c < 0$ translation down
- Try translating the following function

$$y = \frac{-5}{x + 3} - 4$$

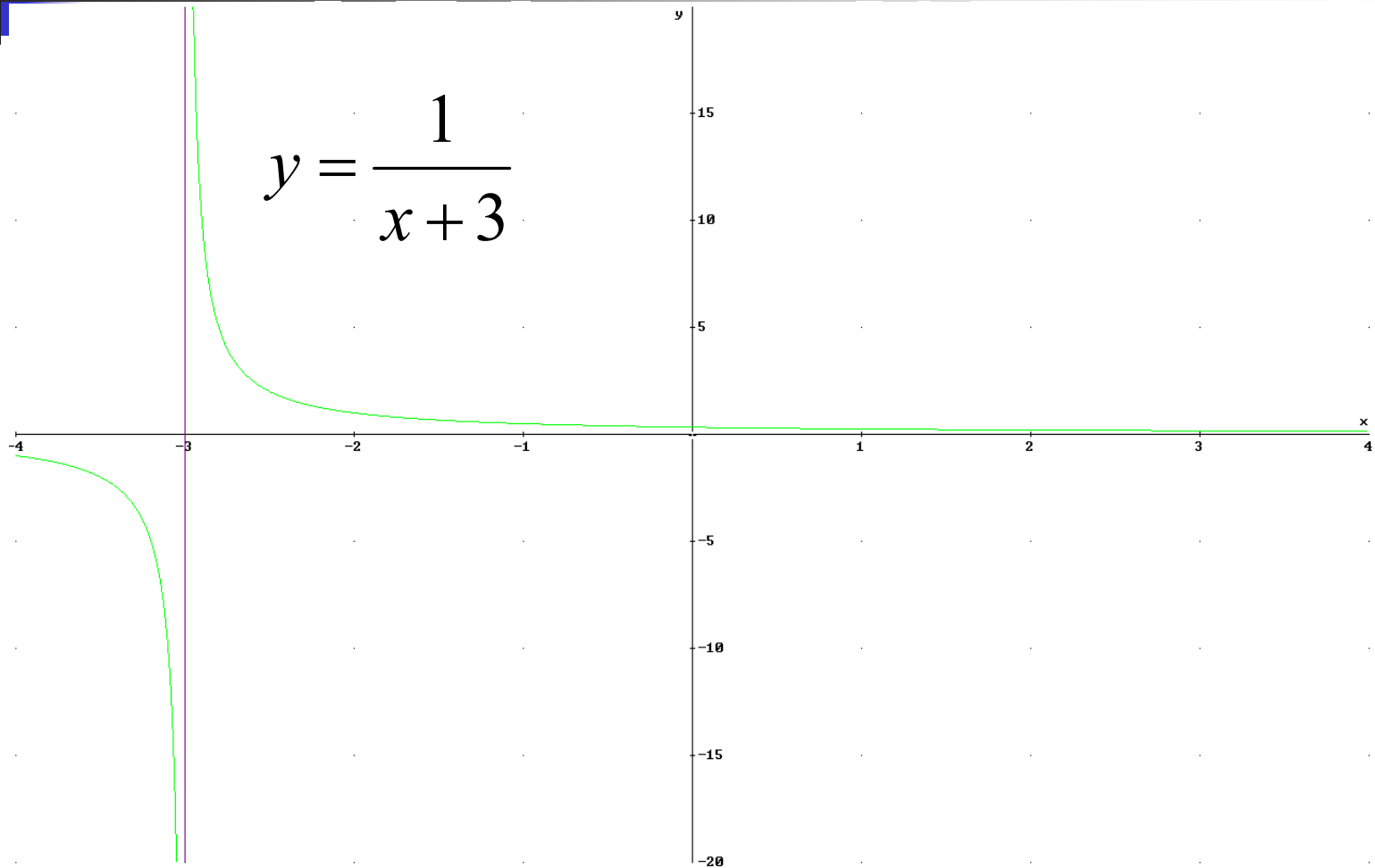
Transformation of Algebraic Functions

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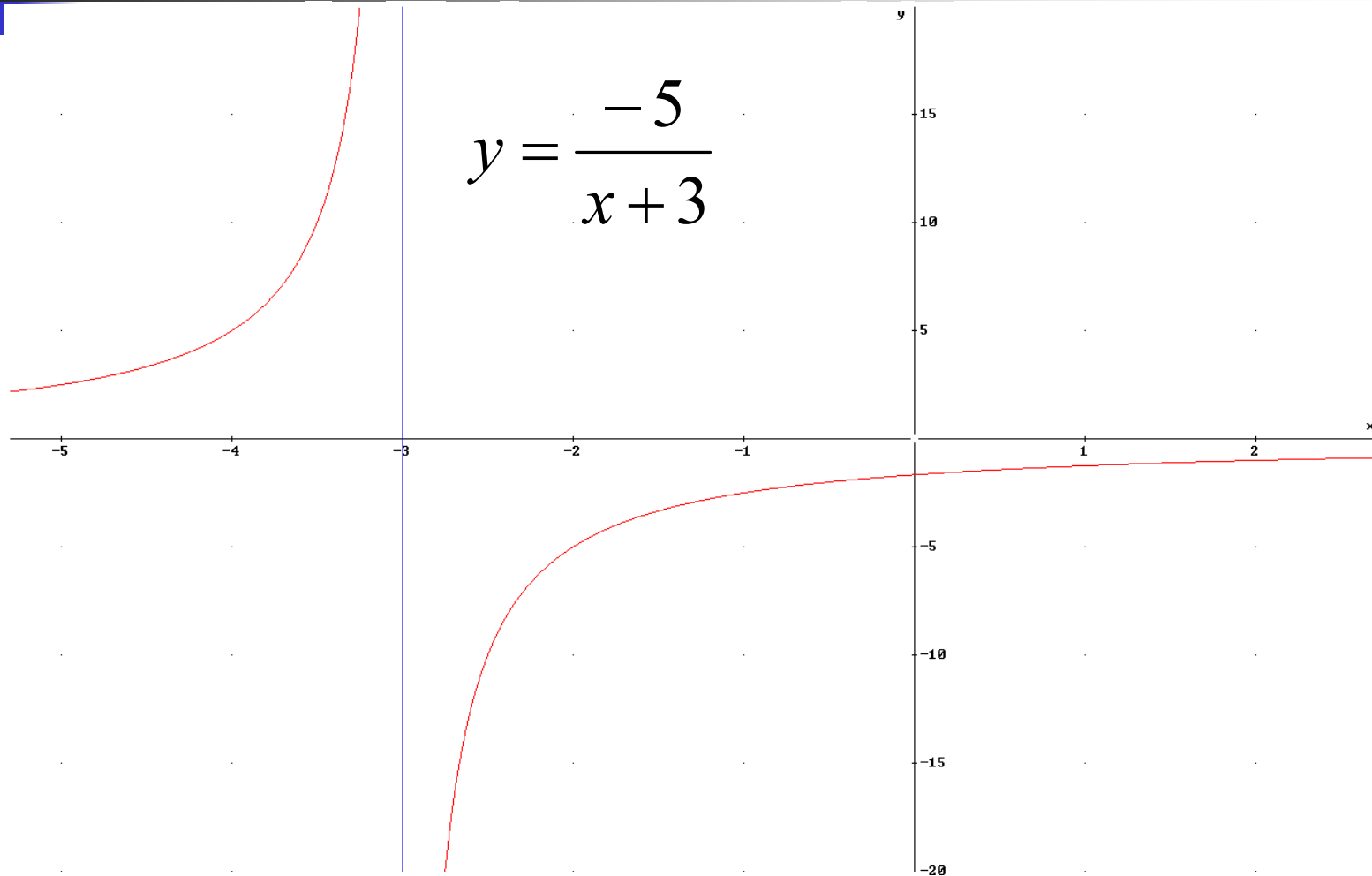
Transformation of Algebraic Functions

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