Rational Functions

- Section 3.5
  - Rational function is the ratio of two polynomial functions

$$r(x) = \frac{n(x)}{d(x)}$$

  - Asymptote comes from combining 3 Greek words "an-sum-piptein" meaning does not fall together with. An asymptote is a curve that another curve approaches but does not ultimately cross.

Model: Poiseuille's Law

- According to Poiseuille's Law, the resistance to flow of a blood vessel is directly proportional to the length and inversely proportional to the fourth power of the radius of the blood vessel.
  - Use the concept of direct and inverse variation to find a model for this law if when the length of the blood vessel is 12 cm and the radius is 0.2 cm the resistance is 25

Direct and Inverse Variation

- Direct Variation: y varies directly as x if there exists a nonzero real number k such that $y = k \cdot x$ (thus the quotient y/x is constant)
- Inverse Variation: y varies inversely as x if there exists a nonzero real number k such that $y = k/x$ (thus the product xy is constant)
- Joint Variation: any mixture of direct and inverse variation

Model: Poiseuille's Law

- According to Poiseuille's Law, the resistance to flow of a blood vessel f is directly proportional to the length l and inversely proportional to the fourth power of the radius r of the blood vessel.
  - Solution:

$$f = \frac{k \cdot l}{r^4}$$

  - For l = 12 we get a rational function model

$$f(r) = \frac{k \cdot 12}{r^4}$$

Constant of Variation

- How can we find the constant of variation k using the fact that when r = 0.2 the flow resistance is 25?

$$f(r) = \frac{k \cdot 12}{r^4}$$

  - Solution: Substitute the point (0.2, 25) into the model and solve for k.

$$25 = \frac{k \cdot 12}{0.2^4} \qquad k = \frac{1}{300} \approx 0.003$$

Rational Function Model

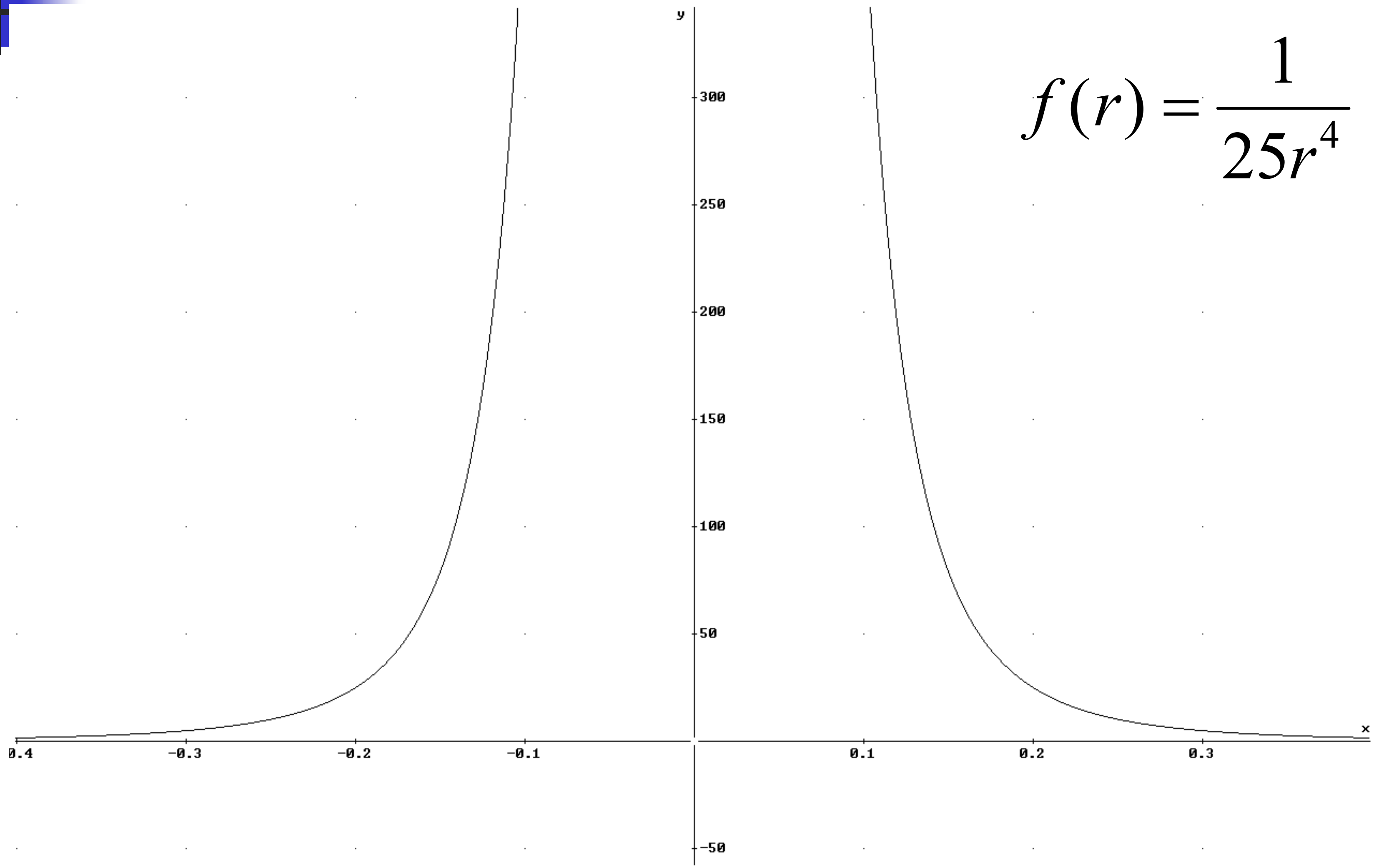
- Poiseuille's Law is a rational function

$$f(r) = \frac{1}{25r^4}$$

- What are the characteristics of a rational function model that make it different then a polynomial or radical model?
  - Concavity or Increasing/decreasing – no
  - End behavior – yes, a rational function may not approach positive or negative infinity as x grows large
  - Continuous – yes, a rational function may have holes or gaps called discontinuities

Poiseuille's Law

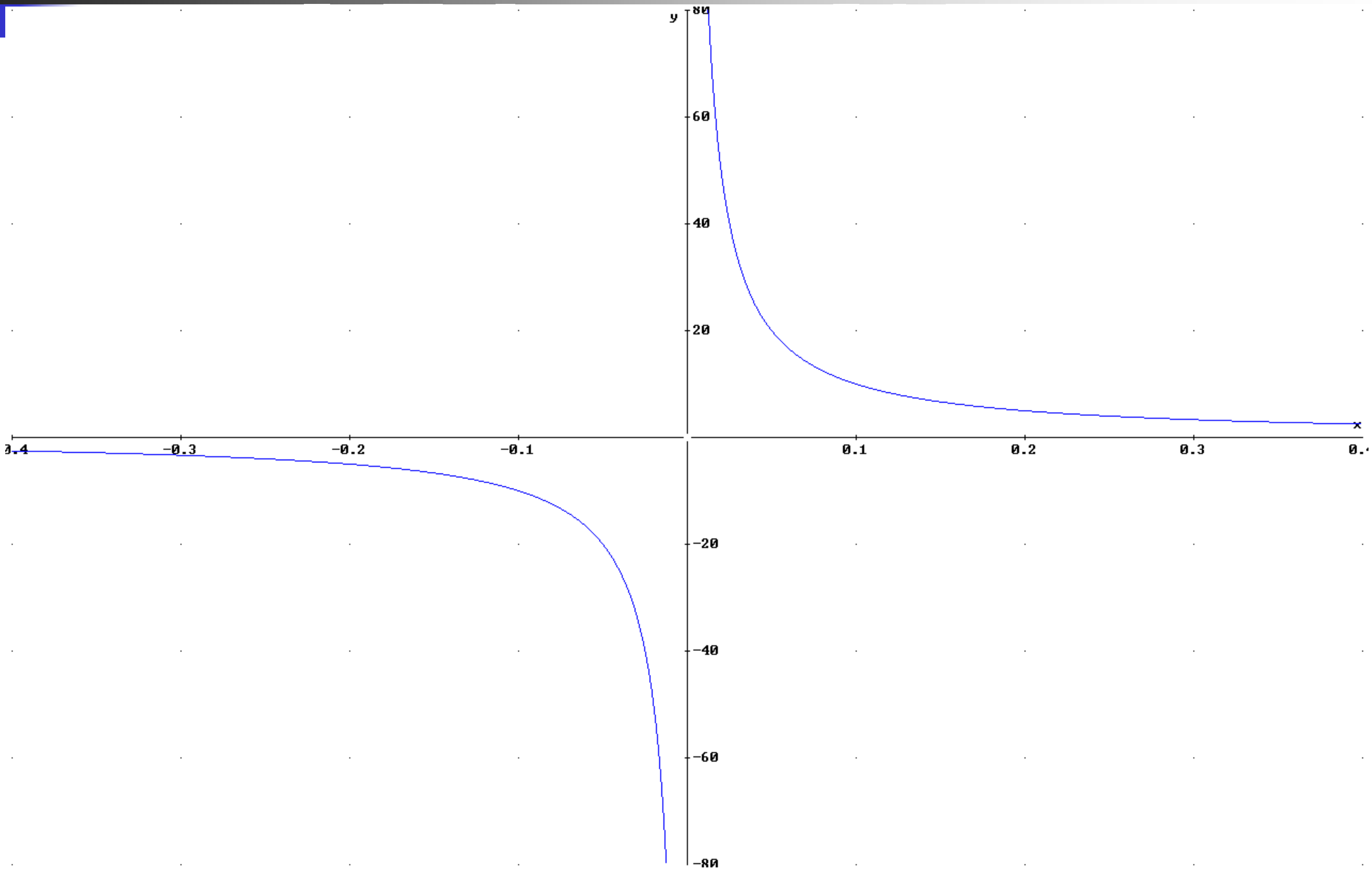
$$f(r) = \frac{1}{25r^4}$$



Basic Rational Function $f(x) = \frac{1}{x}$

- Intermediate Behavior
  - Includes x-intercepts, y-intercept, turning points, and points where function does not exist
- Vertical Asymptote: a line $x = a$ for which the graph goes to negative or positive infinity as x approaches a
  - What is the vertical asymptote for the basic rational function?
  - How can we find the vertical asymptotes for any rational function?

Basic Rational Function $f(x)=\frac{1}{x}$



Basic Rational Function $f(x)=\frac{1}{x}$

- End Behavior
  - Approaches an asymptote which may or may not be linear
  - Differs from polynomial or radical models – may not tend to infinity
- End Behavior Asymptote: a line or a curve which the graph of the rational function approaches as x approaches positive or negative infinity
  - What is the horizontal asymptote for the basic rational function?
  - How can we find a horizontal asymptote if one exists?

End Behavior Model

- End Behavior Model for Rational Function
  - As x approaches infinity the largest degree terms in the numerator and denominator account for most of the change
  - End Behavior Model is the rational expression formed by taking only the largest degree term from the numerator and denominator
  - End behavior model and the original function will have the same general end behavior

End Behavior Model $y = \frac{ax^n}{bx^m}$

- 3 basic end behavior models
- n < m Example: $y = 2/3x^2$
  - What is the only possible end behavior?
- n = m Example: $y = 7x^3/5x^3$
  - What is the end behavior for this case?
- n > m Example: $y = 8x^4/3x^3$
  - What is the end behavior if n = m+1?
  - What is the end behavior if n is more than one degree larger than m?

Complete Graph of Rational Function

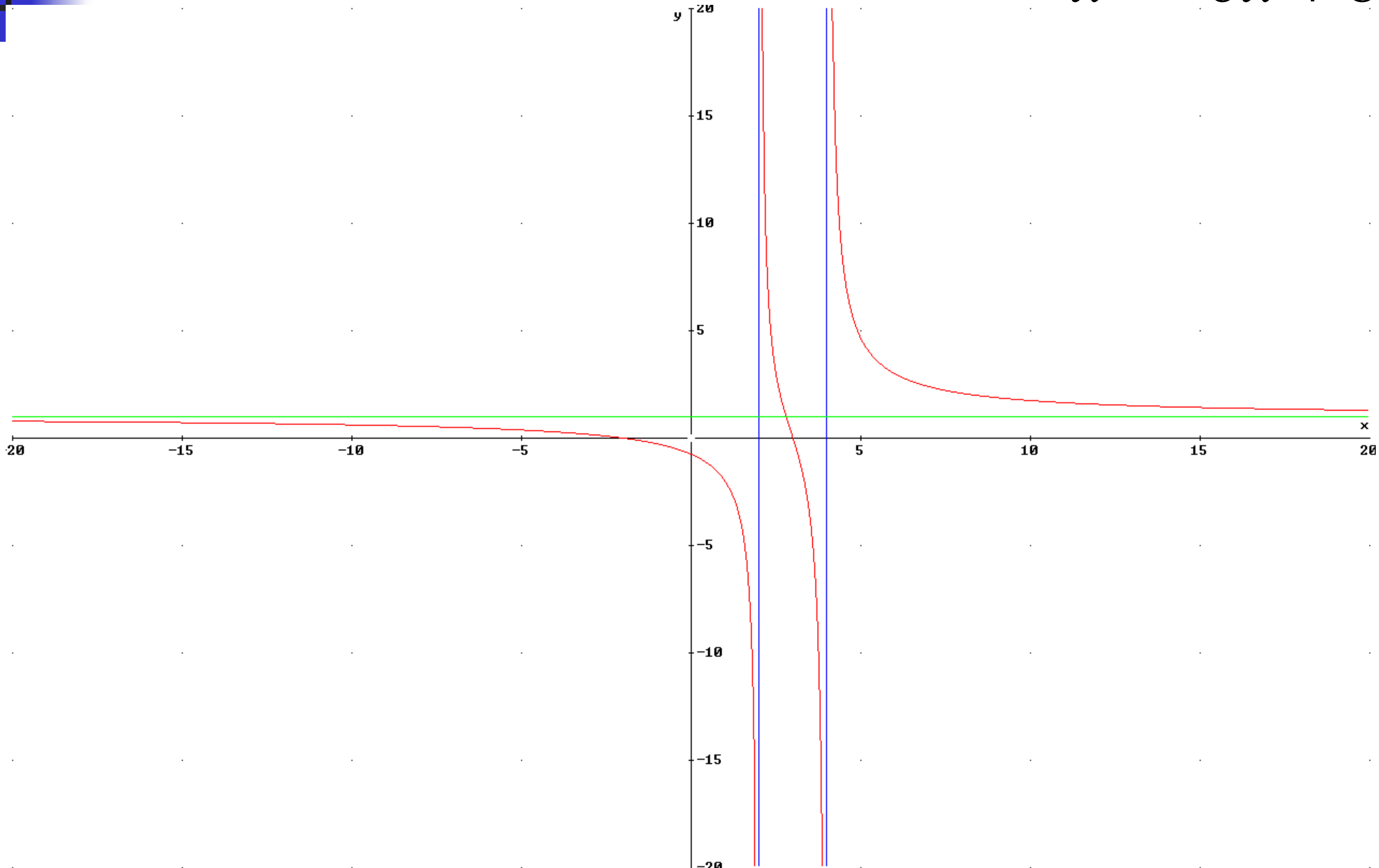
- Use the algorithm described in Assessment 3 and Example 4 to determine a complete graph of the following function (factored form provided)

$$f(x) = \frac{x^2 - x - 6}{x^2 - 6x + 8}$$

$$f(x) = \frac{(x+2)(x-3)}{(x-2)(x-4)}$$

Complete Graph

$$f(x)=\frac{x^2-x-6}{x^2-6x+8}$$



Class Participation Activity

- If the denominator is zero for x = a, then is x = a always a vertical asymptote? Try the following example.

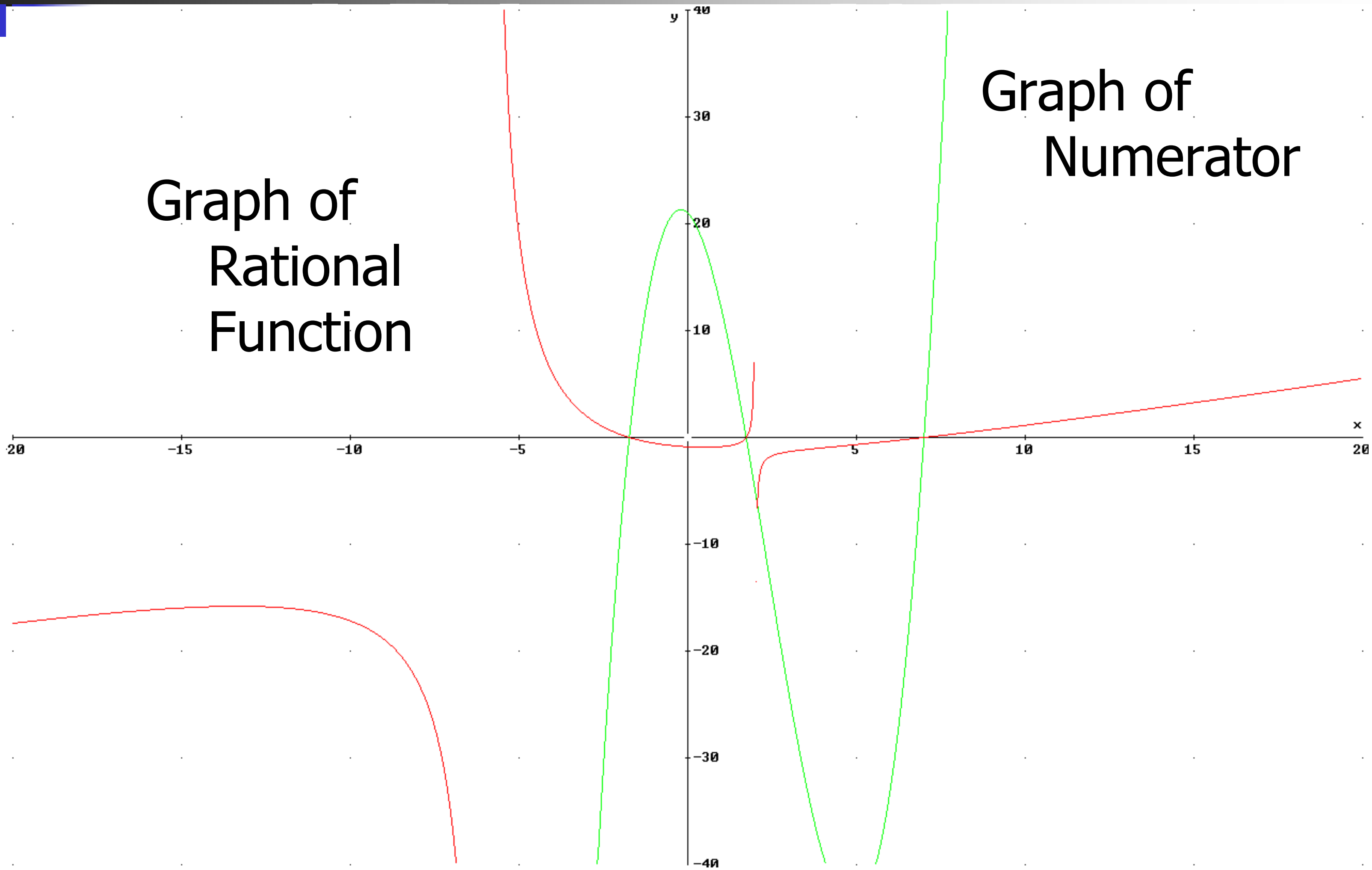
$$f(x) = \frac{x-1}{x^2-1}$$

Complete Graph of Rational Function

- Use the algorithm described in Assessment 3 and Example 5 to determine a complete graph of the following function which does not factor easily

$$g(x) = \frac{x^3 - 7x^2 - 3x + 21}{2x^2 + 8x - 24}$$

Graph of Numerator to Approximate x-intercepts



Complete Graph