Radical Functions

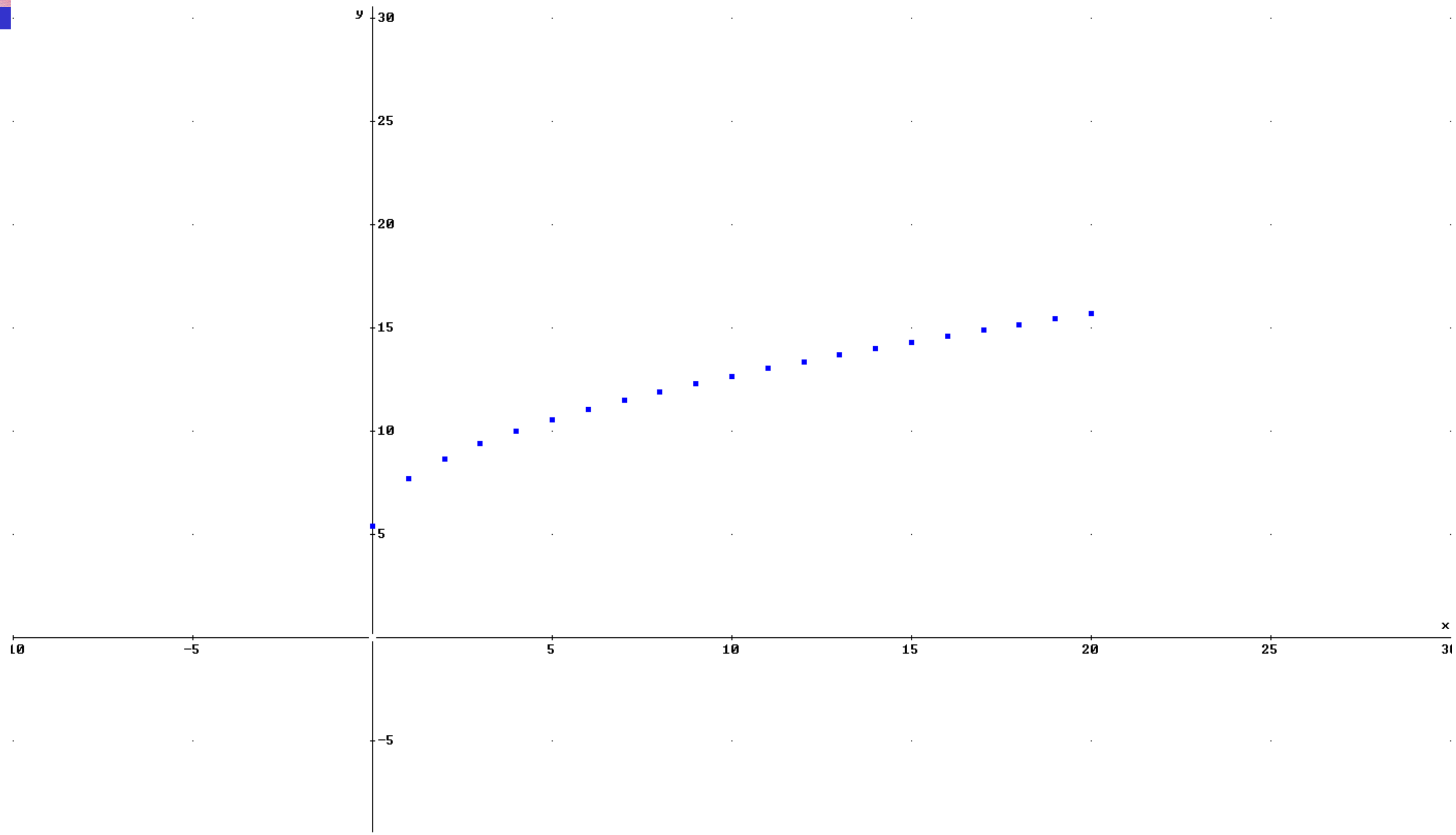
- Section 3.4
  - Modeling continuous relationship which is increasing and concave down

Model: Cost Function

- A company manufactures a quantity q of a product per month. They invest $2.3 million in capital towards technology (machinery, computers, etc.) and a fixed cost investment of $5.4 million. Data is collected on the cost for varying quantities q (in 100,000 units).
  - Determine a model for cost.

Q	C
0	5.4
1	7.7
2	8.65
3	9.38
4	10
5	10.54
6	11.03
7	11.49
8	11.91
9	12.3
10	12.67

Cost Data



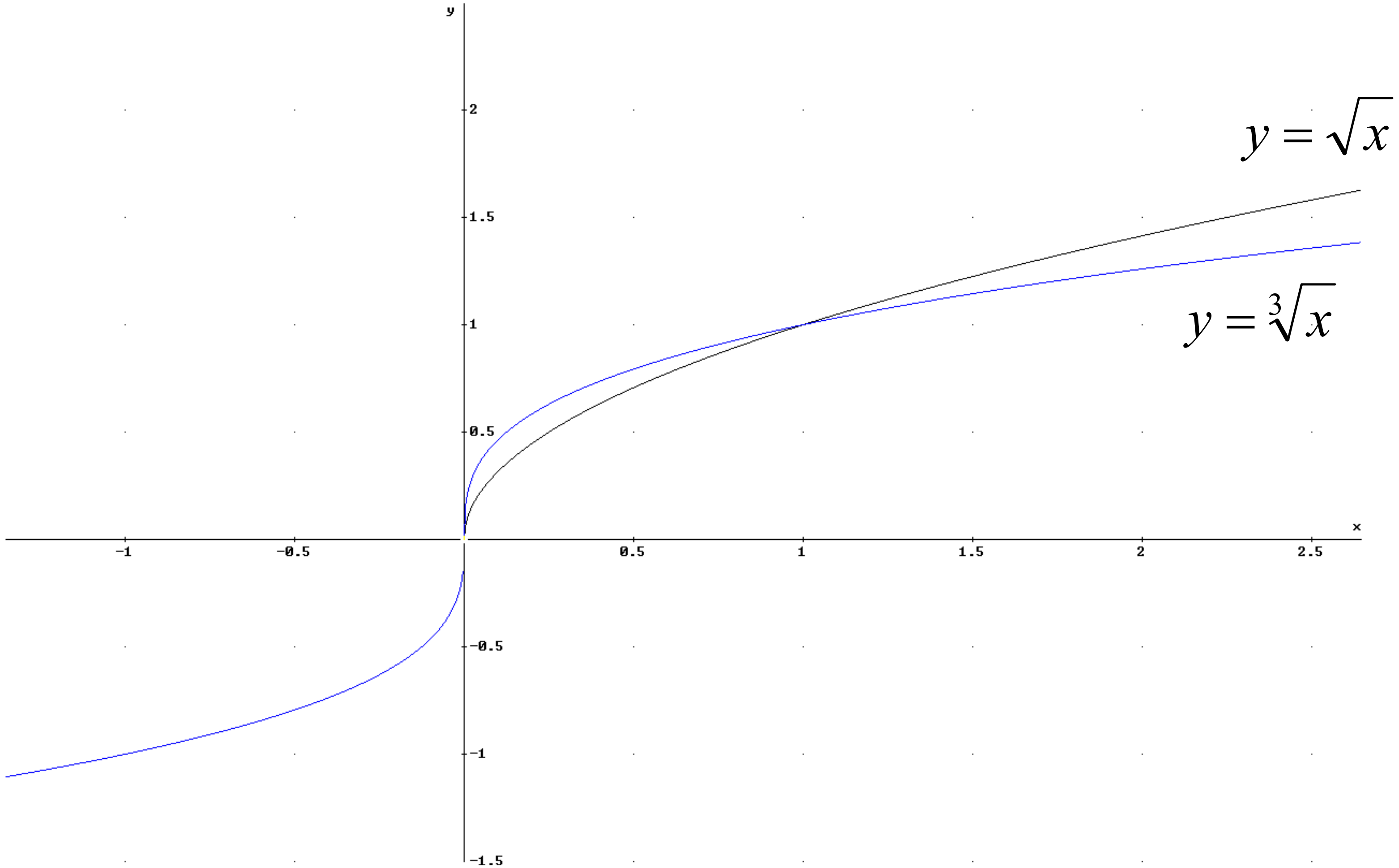
Properties of Model

- Is the data increasing or decreasing?
  - Cost increases as production increases
- Is the data concave up or concave down?
  - Data is concave down
- Use the Ladder of Powers to determine a model with these characteristics. Can it be a polynomial model?
  - No, polynomial functions are either increasing concave up or decreasing concave down

Radical Functions

- Function of form $r(x) = \sqrt[n]{p(x)}$
  where p(x) is a polynomial, integer n>1
  - If n is even, what is the domain of r(x)?
  - If n is odd, what is the domain of r(x)?
  - What is the concavity of a radical power function $y = ax^n$ when $0 < n < 1$?

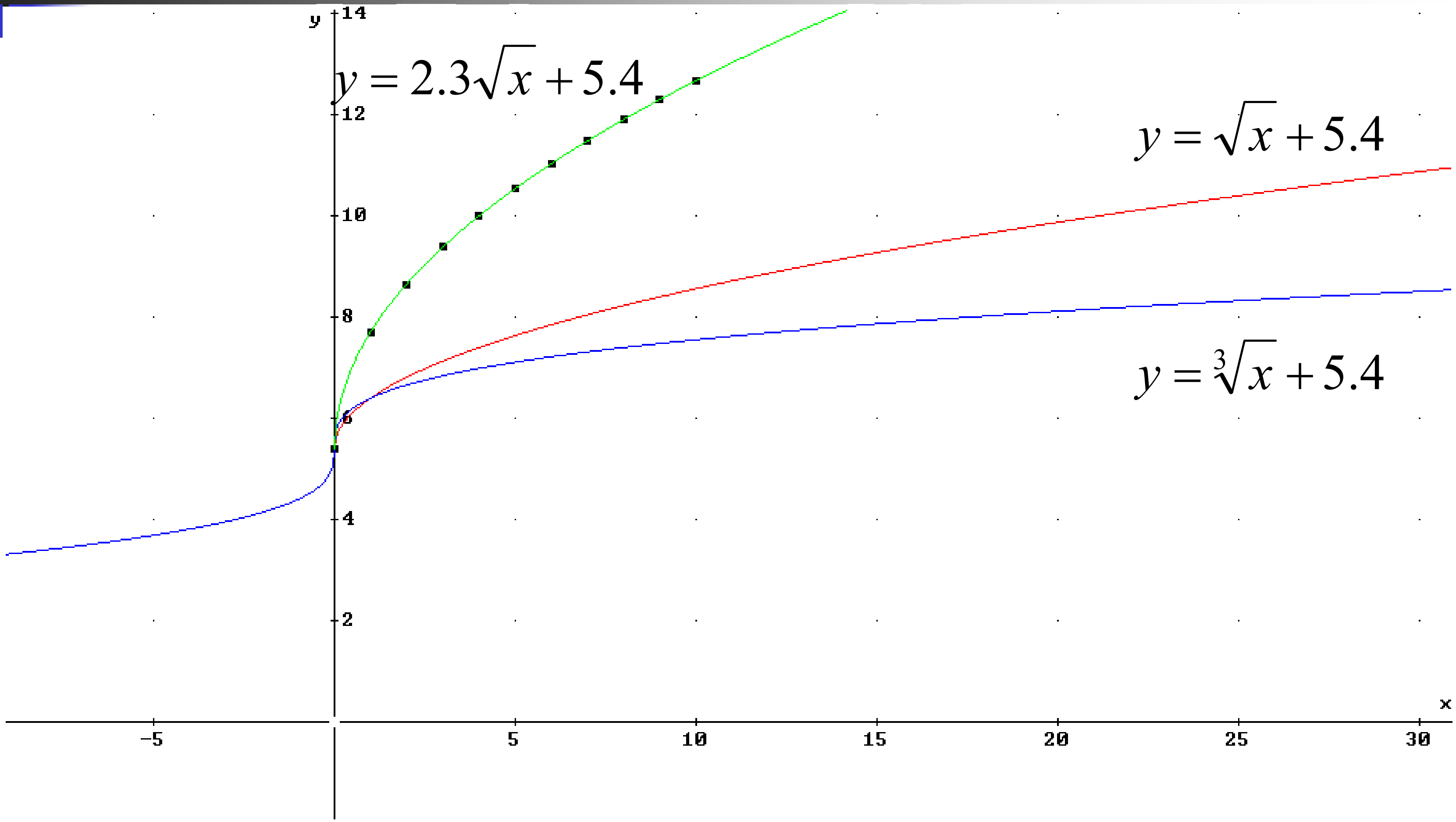
Radical Function: index 0<n<1



Determining Radical Model

- Ladder of Powers to decide if n = ½,
  n = 1/3, n = ¼ etc.
- Graph $y = x^{1/3}+5.4$ and $y = x^{1/2}+5.4$, which is a better fit to the Cost Data?
- Use Derive or a graphing calculator to fit a radical function to the data
  - Solution: $C(q) = 2.3\sqrt{q} + 5.4$
  - Cobb-Douglas Production Function

Cost Model


y
14
12
10
8
6
4
2
x
-5
5
10
15
20
25
30
$y = 2.3\sqrt{x} + 5.4$
$y = \sqrt{x} + 5.4$
$y = \sqrt[3]{x} + 5.4$

Solving Radical Equations - Algebraic Method

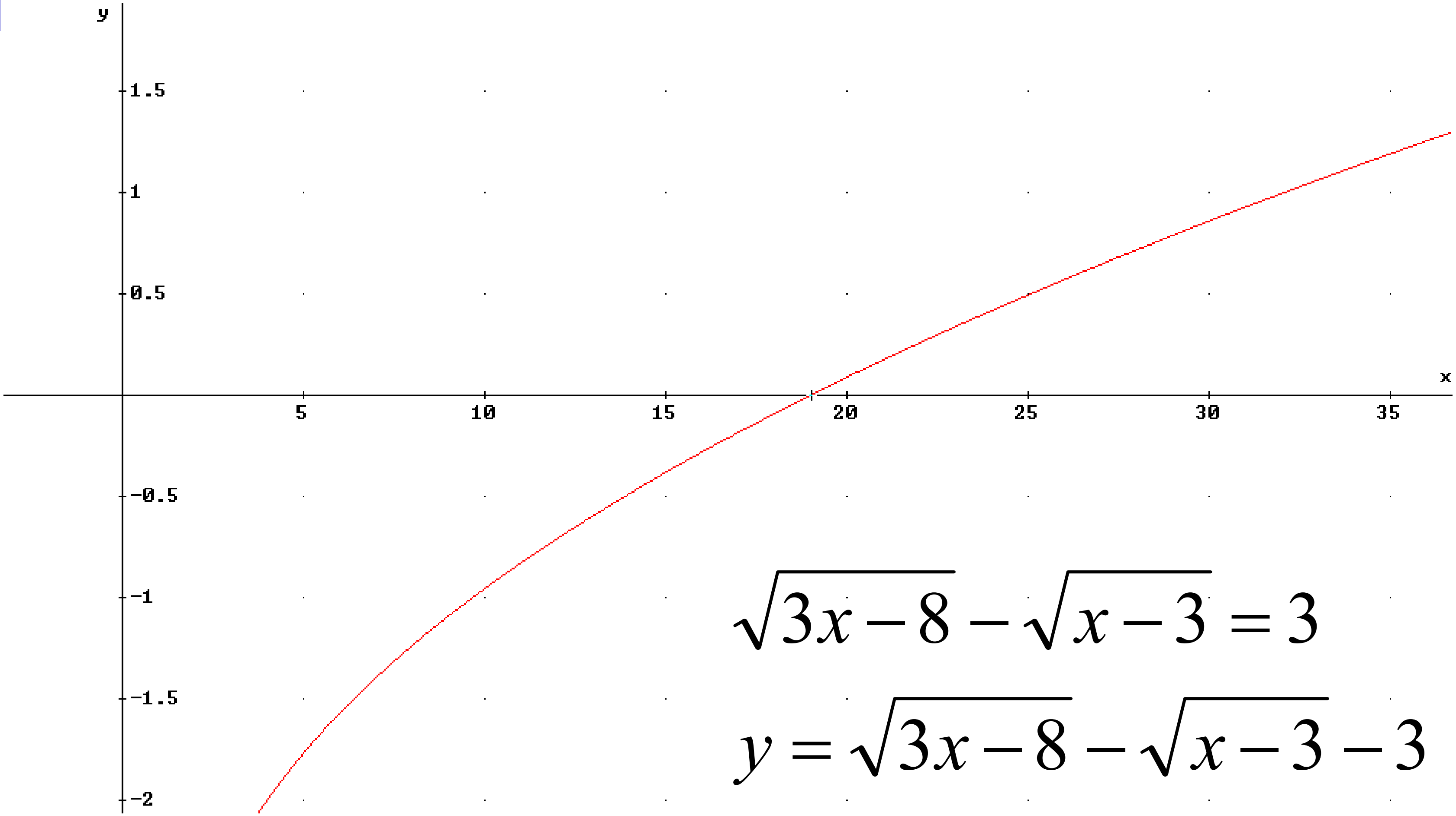
- Isolate the radical expression
- Undo the nth root by using the inverse function – the nth power
- For which nth roots do we need to check for extraneous roots?
  - If nth root is even, taking an even power can result in extraneous roots
  - If nth root is odd, taking an odd power will not result in extraneous roots

Double Radical Equation

- Isolate one of the radical expressions
- Take the square to remove one of the square roots
- Isolate the remaining radical and square again
- Check for extraneous roots
  - Solve using algebraic and graphic method

$$\sqrt{3x-8}-\sqrt{x-3}=3$$

Double Radical Equation

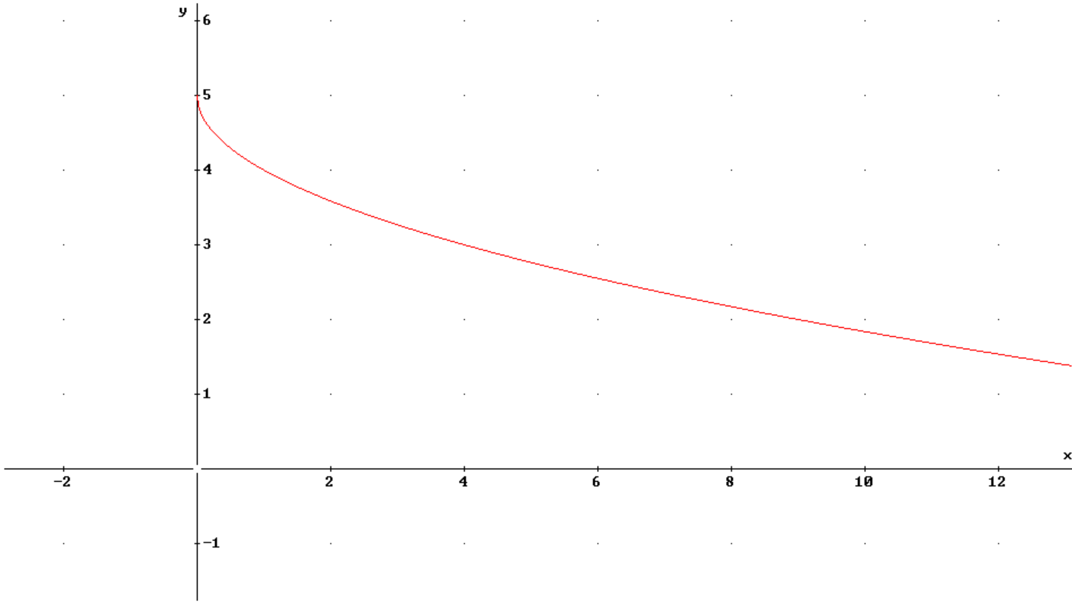


$$\sqrt{3x-8}-\sqrt{x-3}=3$$

$$y=\sqrt{3x-8}-\sqrt{x-3}-3$$

Class Participation Exercise

1. Determine what type of function has the given graph? Is it polynomial or radical? Why?



2. Solve the following radical equation:

$$\sqrt{2x+5}+\sqrt{x+2}=5$$

Linearization

- Use radical function to make nonlinear polynomial data linear
- Model the linear data
- Convert linear model into polynomial model

X	Y
0	9
1	5
2	22
3	75
4	179
5	349
6	600
7	947
8	1405
9	1989
10	2714

Linearization

- Use Derive or graphing calculator to plot data and transformed data

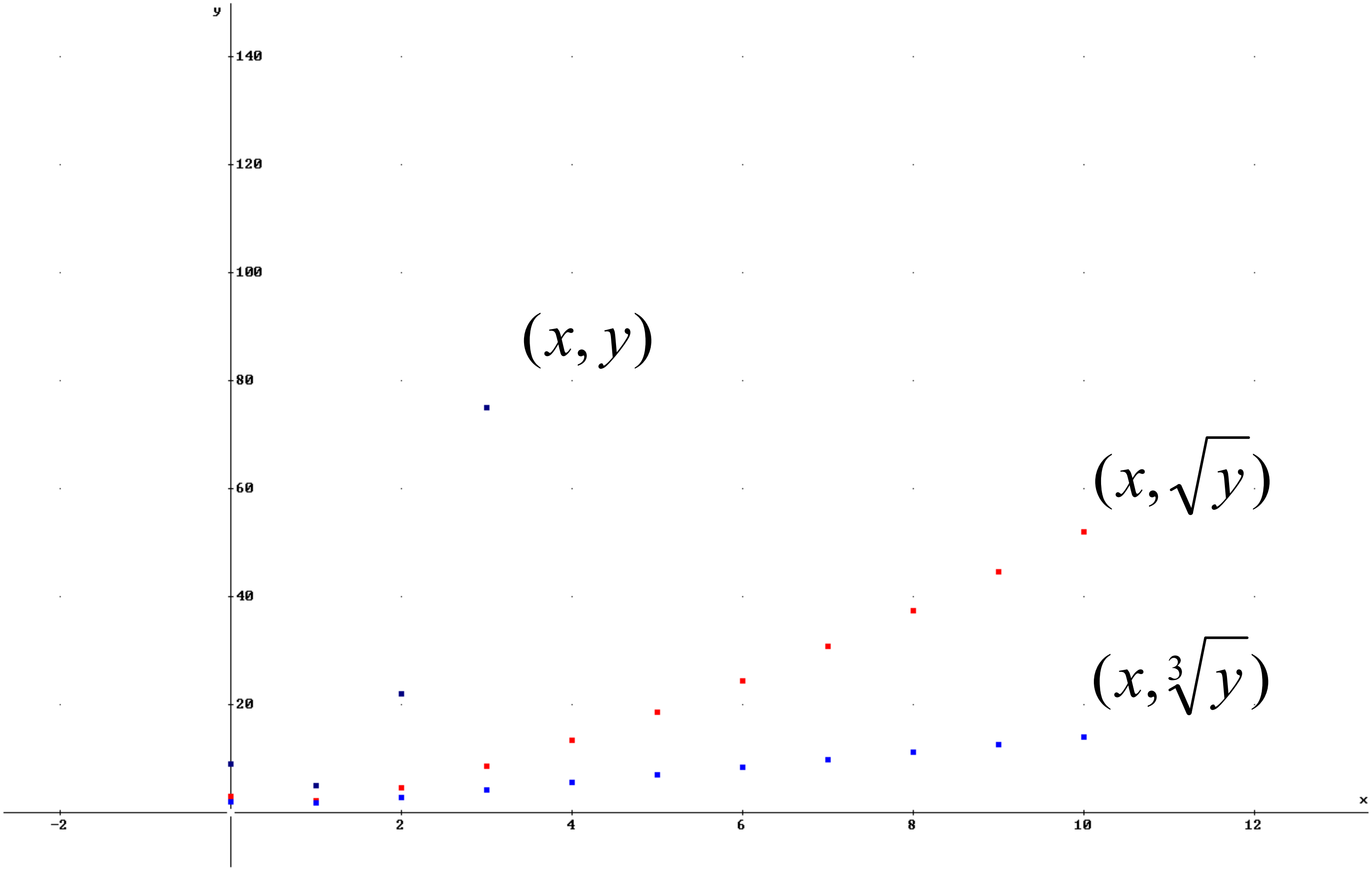
$$(x, y) \qquad (x, \sqrt{y}) \qquad (x, \sqrt[3]{y})$$

- Which of these makes the data linear?

Linearization – Transformed Data

(x, y)		$(x, \sqrt{y})$		$(x, \sqrt[3]{y})$	
0	9	0	$\sqrt{9}$	0	$9^{1/3}$
1	5	1	$\sqrt{5}$	1	$5^{1/3}$
2	22	2	$\sqrt{22}$	2	$22^{1/3}$
3	75	3	$\sqrt{75}$	3	$75^{1/3}$
4	179	4	$\sqrt{179}$	4	$179^{1/3}$
5	349	5	$\sqrt{349}$	5	$349^{1/3}$
6	600	6	$\sqrt{600}$	6	$600^{1/3}$
7	947	7	$\sqrt{947}$	7	$947^{1/3}$
8	1405	8	$\sqrt{1405}$	8	$1405^{1/3}$
9	1989	9	$\sqrt{1989}$	9	$1989^{1/3}$
10	2714	10	$\sqrt{2714}$	10	$2714^{1/3}$

Linearization – Data Plot



Linearization

- Which transformed data is linear?
  - Solution: Cube root, so data is modeled by a cubic function
  - Linearized data model: y = 1.3x + 0.77
  - Actual data model: $y^{1/3} = 1.3x + 0.77$
  - Best fit cubic model:

$$y = 2.5x^3 + 3x^2 - 9.5x + 9$$