Absolute Value and Piecewise Defined Functions

- Section 3.3
  - Piecewise Defined Function involves more than one expression for different parts of the domain.
  - Examples: Absolute Value Function
    Greatest Integer Function

Model: Quality Control

To assure quality of a 1000 hour light bulb the company randomly selects a sample of n = 100 bulbs and measures how long they last. The mean hours the 100 bulbs last is m = 1002 hours. Quality control wants the mean to be within 3 sigma control units which is defined as

$\frac{3s}{\sqrt{n}}$ where s = 10.5 is the standard deviation.

- Find a model for the quality control process.

Quality Control

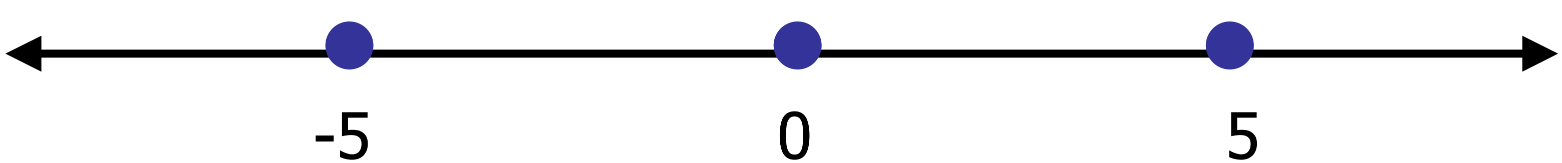
- Quality control is an example of a confidence interval problem
  - Investor wants stock sold if it is $25 per share or more from $100 per share
  - An opinion poll has a margin of error of 2.5% from 52%
- Absolute value function can be used to model confidence interval problems

Absolute Value - Geometric

- The nondirected distance from the origin 0 to a point on the real number line p is represented by the $|x| = p$.

  - Example: Let the point $p = 5$.

Then $|x| = 5$ is the set of points at a distance of 5 from 0. So $x = 5$ or $x = -5$.



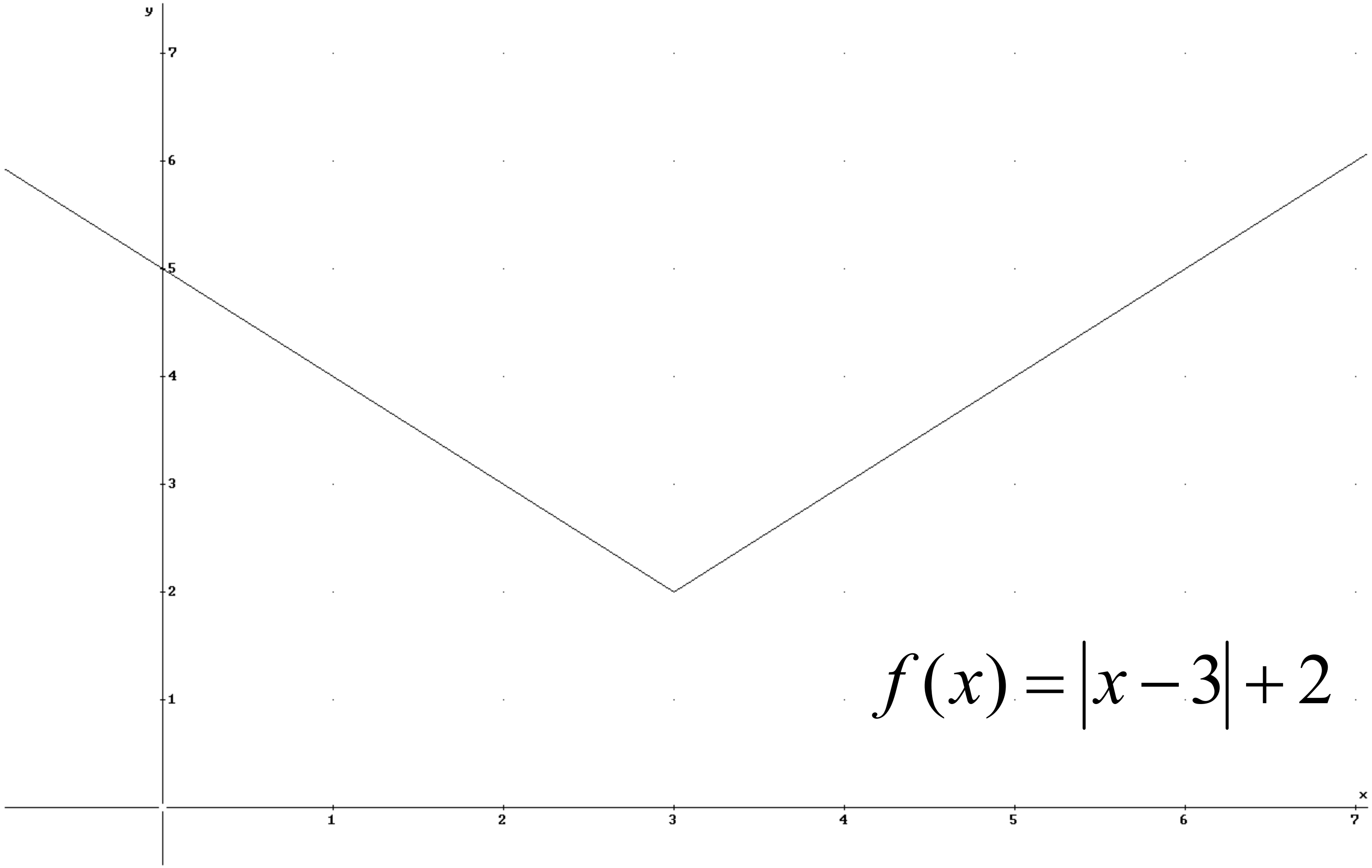
Absolute Value - Algebraic

- The geometric definition leads to the following algebraic definition

$$f(x) = \begin{cases} x & if \;\; x \geq 0 \\ -x & if \;\; x < 0 \end{cases}$$

- What is the domain and range of f(x) = |x| ?
- What would the complete graph of an absolute value function have to include?

Absolute Value Function



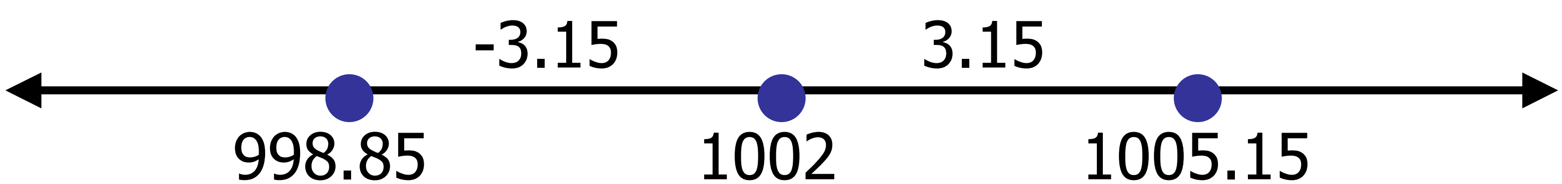
$$f(x) = |x - 3| + 2$$

Quality Control Model

- The mean hours the bulbs last is m = 1002 hours. Quality control wants the mean to be within

$$\frac{3s}{\sqrt{n}} = \frac{3(10.5)}{\sqrt{100}} \approx 3.15\ hours$$

  - Find an absolute value inequality for all numbers whose distance from m is less than 3.15



  - Solution: $| x - 1002 | < 3.15$ so $998.85 < x < 1005.15$

Solving Linear Absolute Value Equations and Inequalities

- Apply the absolute value rules to reduce the problem to linear cases
- $| ax+b | = p$ is equivalent to
  - $ax+b = p$ or $ax+b = -p$
- $| ax+b | < p$ is equivalent to
  - $ax+b < p$ and $ax+b > -p$ so $-p < ax+b < p$
- $| ax+b | > p$ is equivalent to
  - $ax+b > p$ or $ax+b < -p$

Solving Nonlinear Absolute Value Equations and Inequalities

- Use graphic methods to solve

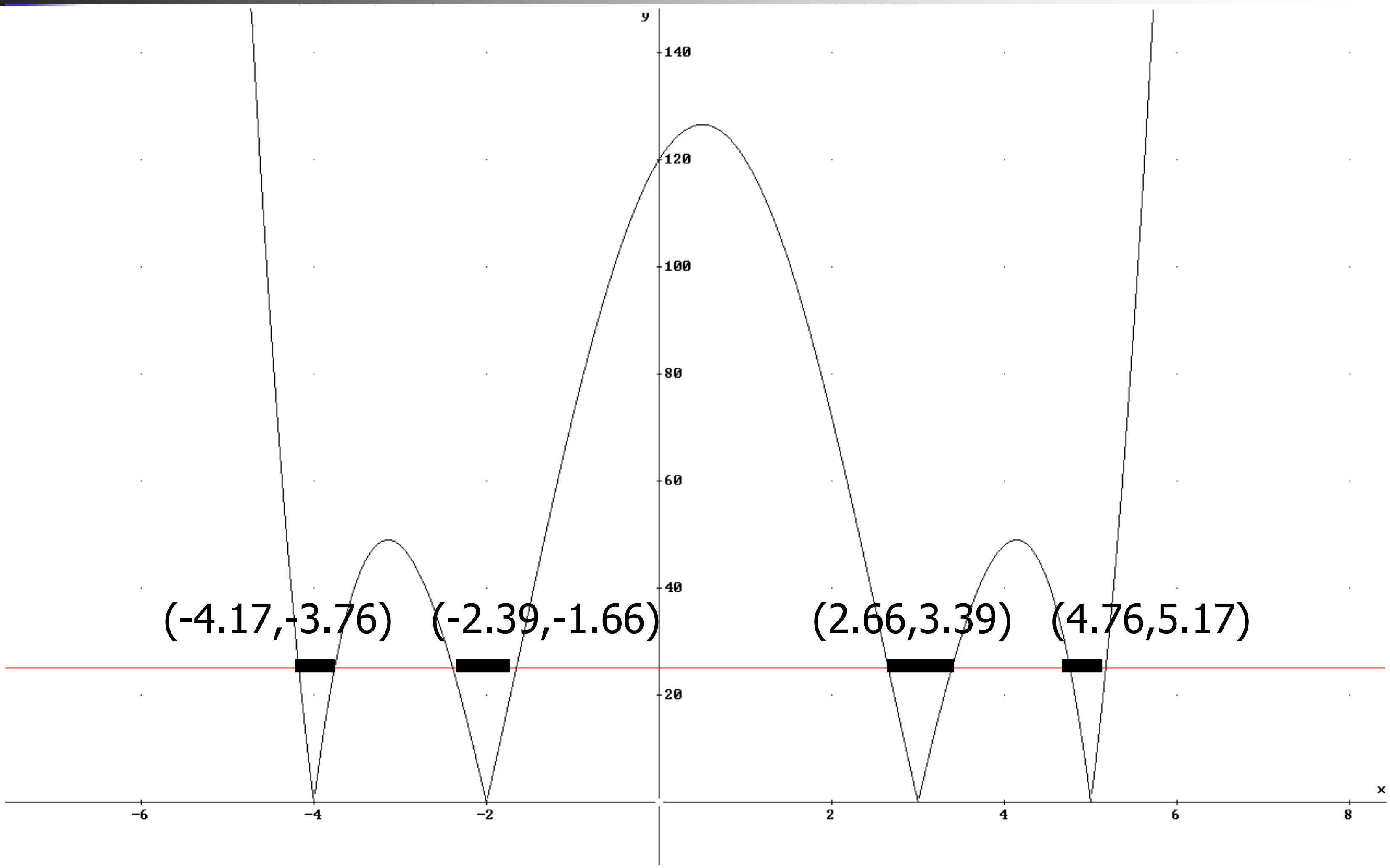
$$\left|x^4 - 2x^3 - 25x^2 + 26x + 120\right| < 25$$

- Graph related function for each side of inequality and approximate intersection

$$y = \left|x^4 - 2x^3 - 25x^2 + 26x + 120\right|$$

$$y = 25$$

Graph of 2 related equations



Solving Nonlinear Graphically

- How can we determine the complete graph of the absolute value function?
  - Find complete graph of function without absolute value, then reflect negative parts in the x-axis

Absolute Value and Radicals

- There is a direct relationship between absolute value and radicals

$$|x| = \sqrt{x^2}$$

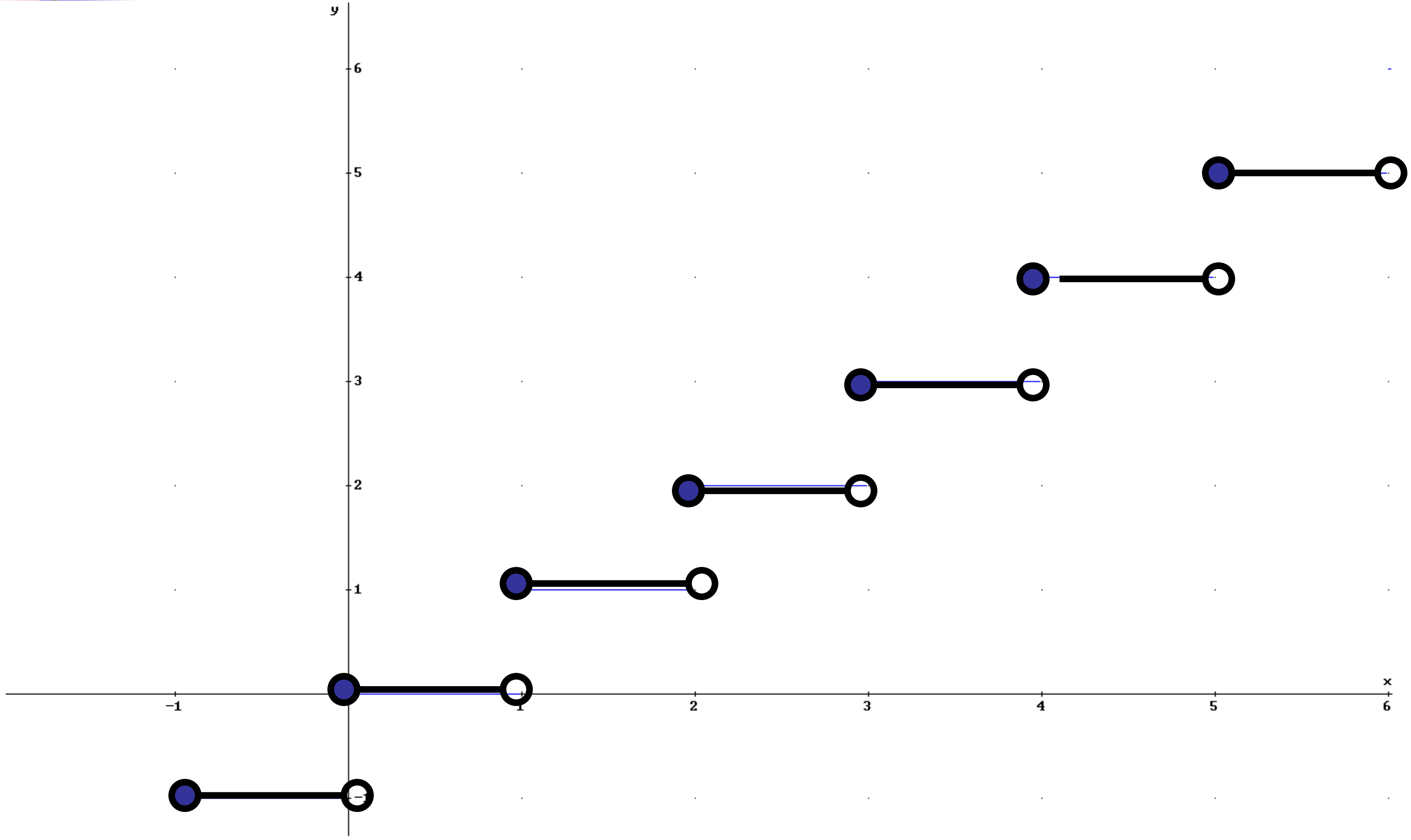
- Use this relationship to convert radical equations into absolute value equations
  - Example: $\sqrt{(5-2x)^2} = 8$

Greatest Integer Function

- [x] is the largest integer less than or equal to x
  - What is [2.5]?
  - Let f(x) = [x]. What is the domain? Range?

- Discontinuous function – f(x) = [x] has jump discontinuities

Greatest Integer Function



Model: Postal Rates

- In 1975 the U.S. Post office instituted a varied rate program based on weight. The rate was 10¢ up to one ounce and 9¢ for each additional ounce or fraction of an ounce. Determine a model for the varied rate program.
  - Solution:
  - $r(w) = 0.10 + 0.09\,[w]$