Polynomial Functions

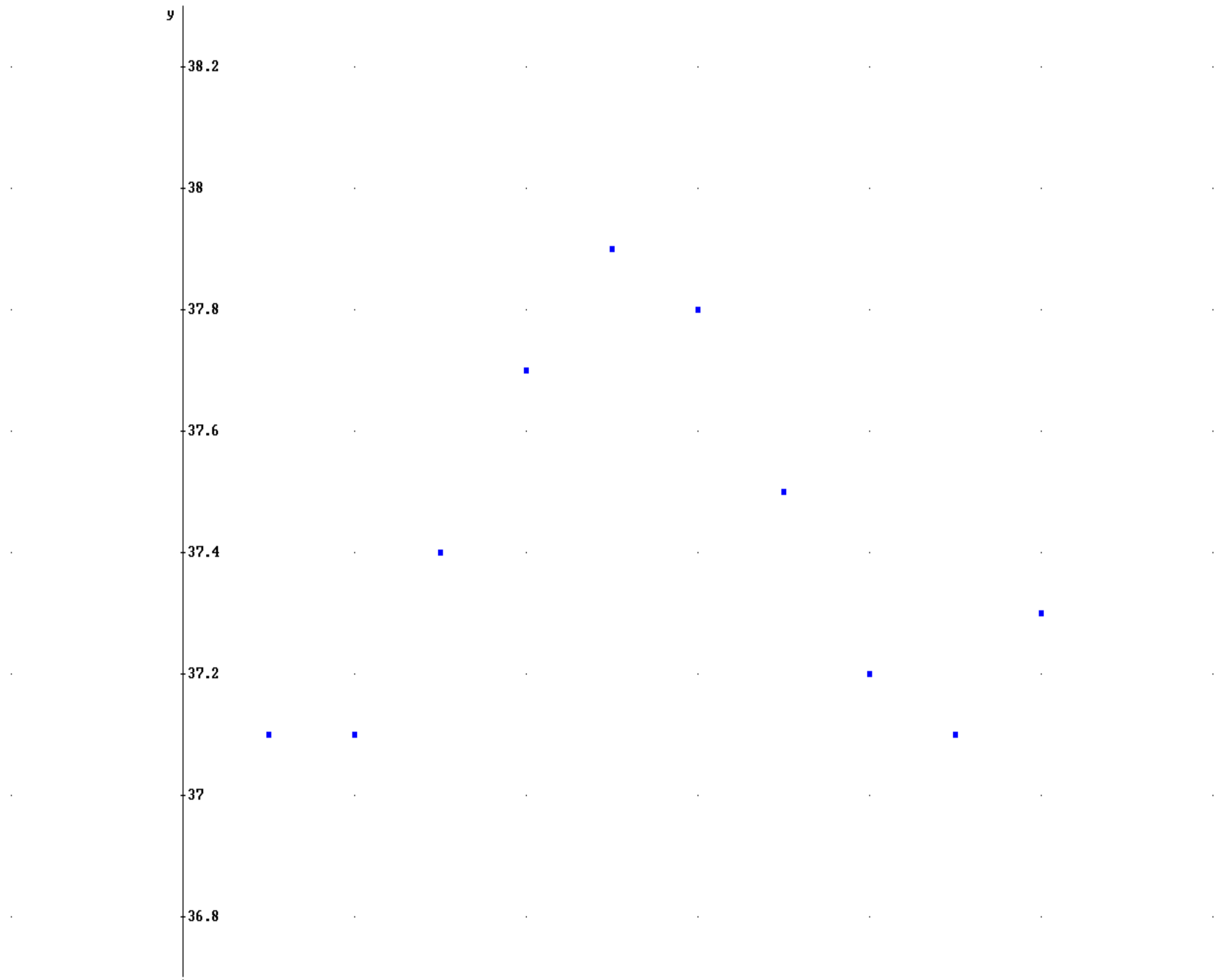
- Section 3.2
  - Polynomial equations of degree 3 or higher have no simple algebraic solution
  - Graphic methods require finding a “complete graph” of the related polynomial function

Model: Biological Rhythms

- Data is collected on normal biological rhythms, such as the sleep-wake cycle, hormone levels, and core body temperatures, in an attempt to find a baseline model to diagnose depression. Core body temperature data was gathered in 10 minute intervals over a 5 day period. A portion of that data is provided.
  - Determine at what time the core body temperature is 37.6 degrees.
  - Is the data linear or curvilinear? Is the model a quadratic, cubic, or higher degree?

10	37.1
20	37.1
30	37.4
40	37.7
50	37.9
60	37.8
70	37.5
80	37.2
90	37.1
100	37.3

Model: Biological Rhythms



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Biological Rhythms

- Best fit model

$$y = 3.9 \times 10^{-7} x^4 - 8.1 \times 10^{-5} x^3 + 0.005 x^2 - 0.1 x + 37.7$$

- Related function for y = 37.6

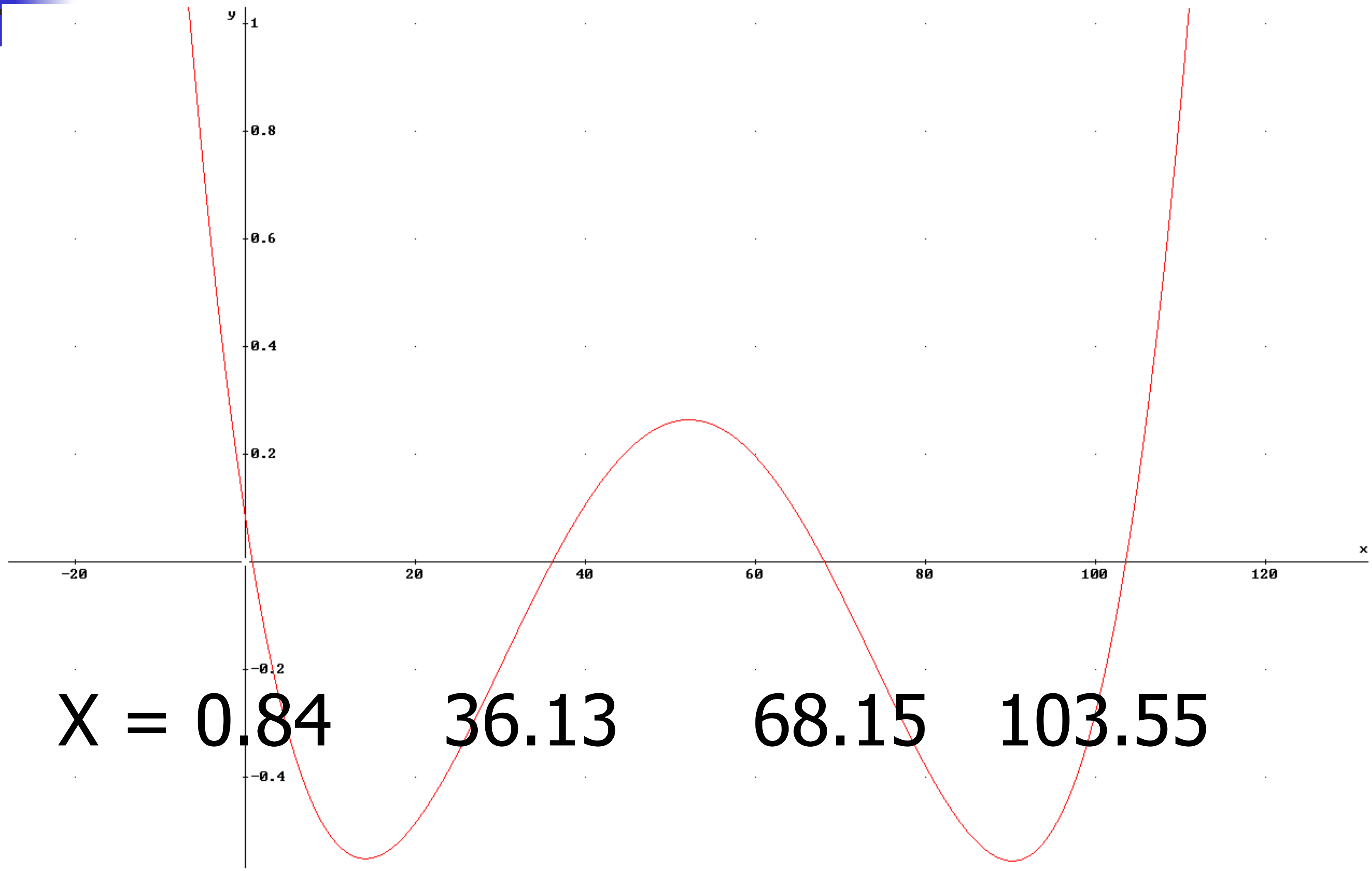
$$37.6 = 3.9 \times 10^{-7} x^4 - 8.1 \times 10^{-5} x^3 + 0.005 x^2 - 0.1 x + 37.7$$

$$y = 3.9 \times 10^{-7} x^4 - 8.1 \times 10^{-5} x^3 + 0.005 x^2 - 0.1 x + 0.1$$

- Use Derive or graphing calculator to approximate the solutions graphically.

Biological Rhythm –

Related Function (coefficients to 9 decimal places)



X = 0.84 36.13 68.15 103.55

Complete Graph

- To solve an equation graphically we must have a complete graph of the related equation, which includes both Intermediate and End Behavior.
- Intermediate Behavior – how the graph behaves near the origin
  - Turns – change in direction
  - Inflection points – change in concavity
  - Intercepts – where graph crosses the x-axis and the y-axis

Intermediate Behavior

- Turns – How can we determine the number of possible turns in a polynomial function using the degree of the function?
- How many x-intercepts can a polynomial function of degree n have?
- How many y-intercepts?
- Explore the following example:

$$y = x^4 - 3x^3 - 28x^2 + 36x + 144$$

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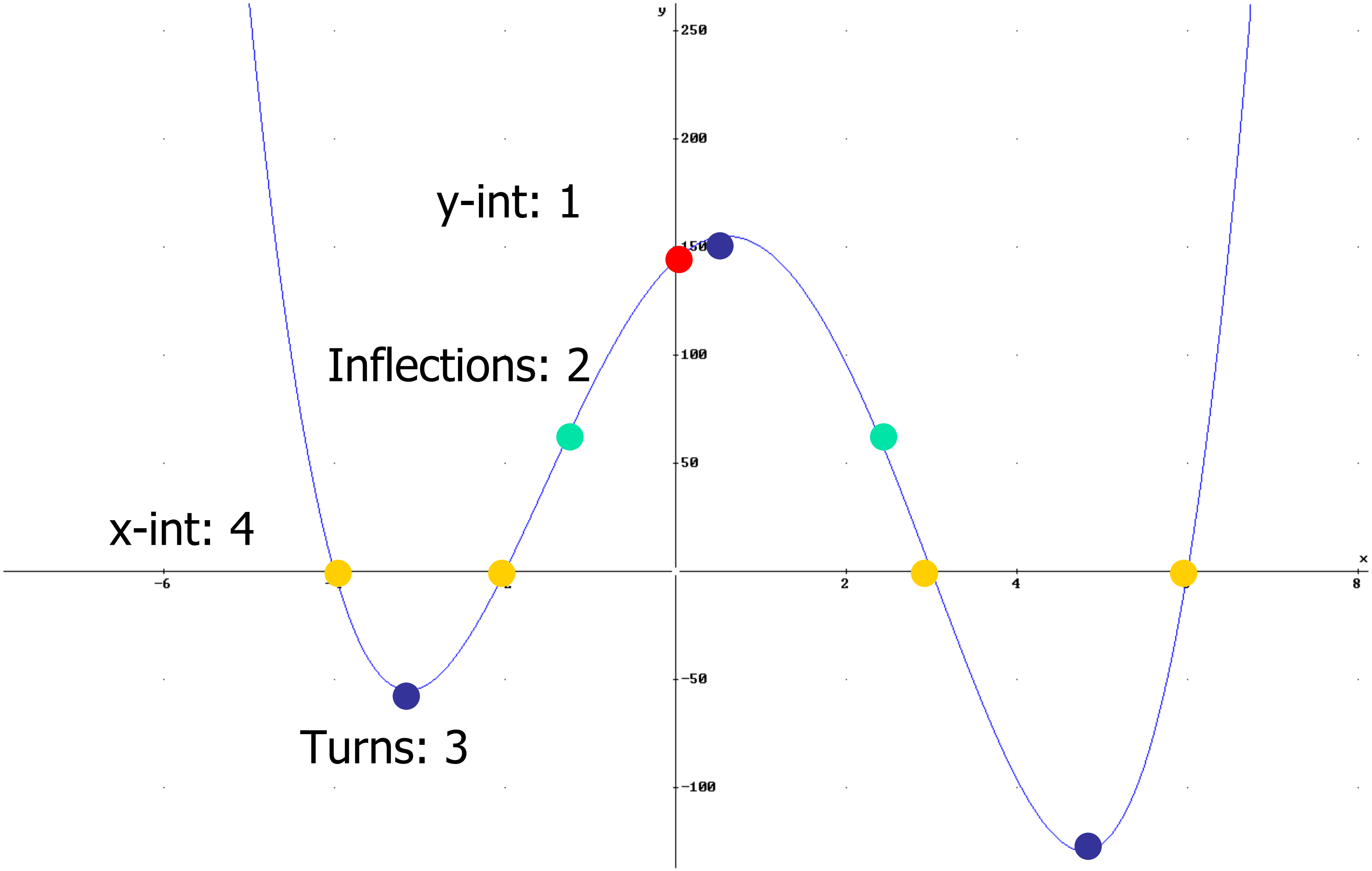
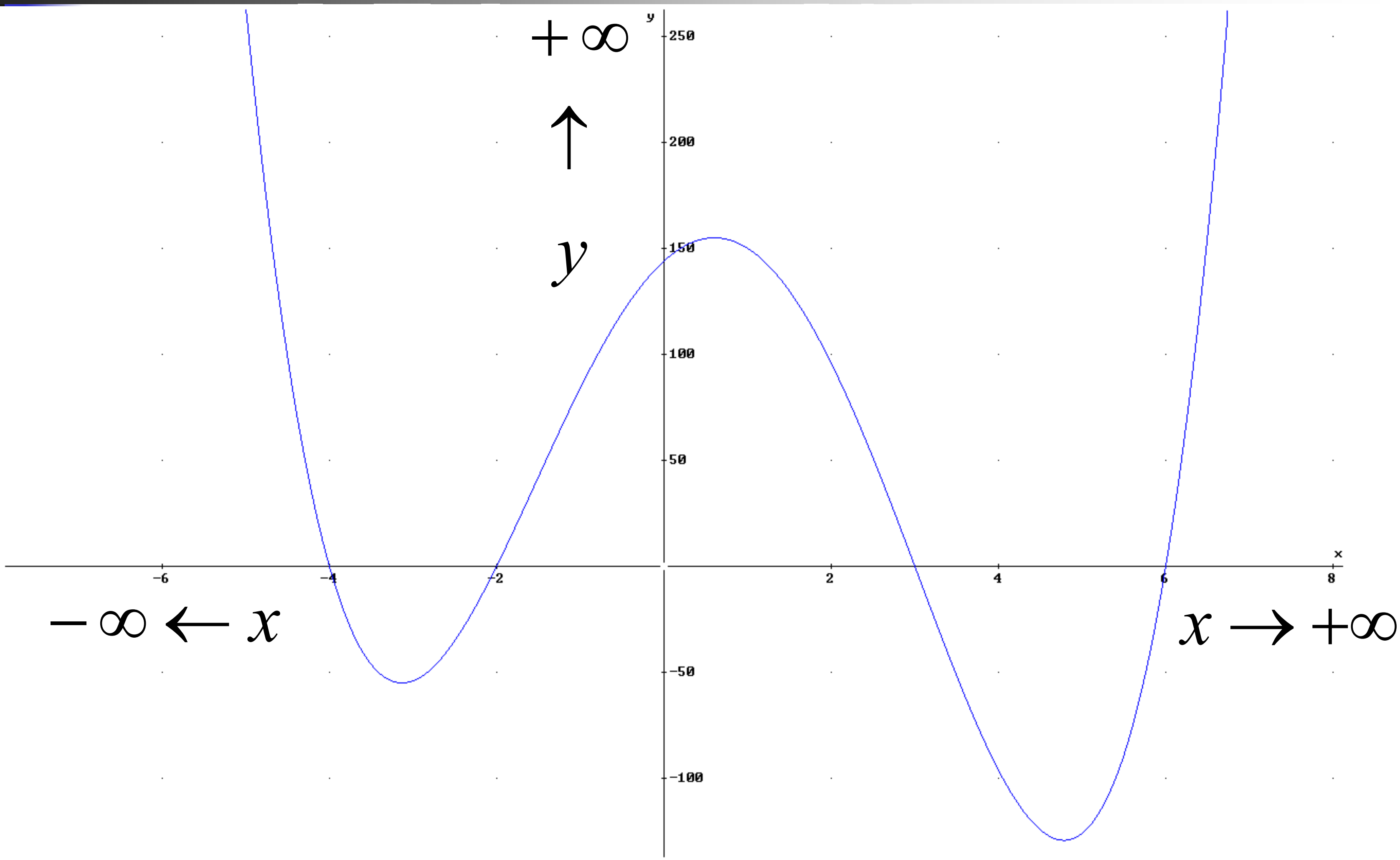


Complete Graph

- End Behavior – how the graph behaves at the extremes
  - What happens as x approaches infinity?
  - What happens as x approaches negative infinity?

$$y = x^4 - 3x^3 - 28x^2 + 36x^2 + 144$$



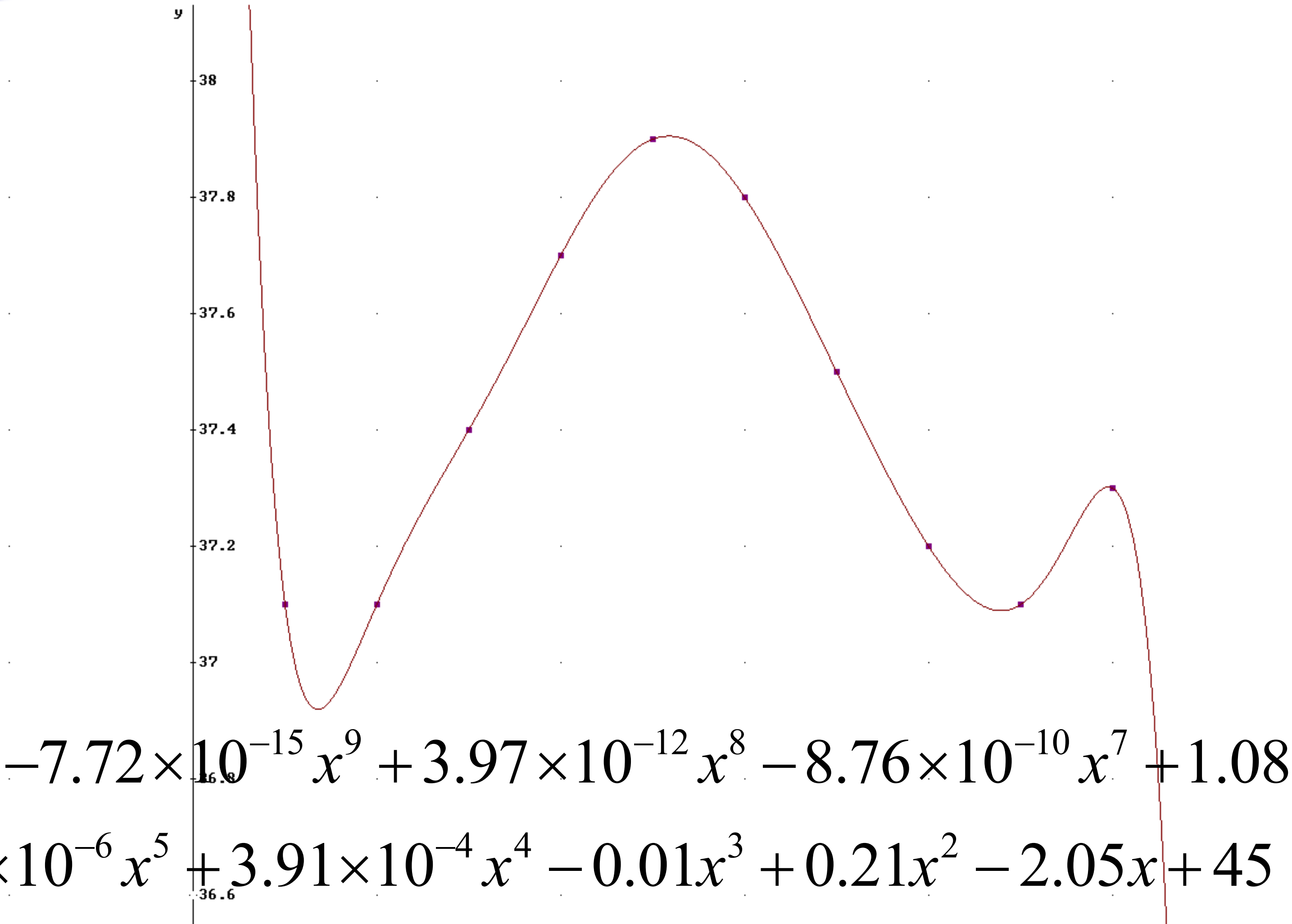
Class Exploration: End Behavior

- Examine the basic polynomial power functions $y = Ax^n$ to explore end behavior.
  - What are the only 4 possible end behaviors?
  - How can we use the leading term of a polynomial to determine the end behavior?

Interpolating Polynomial

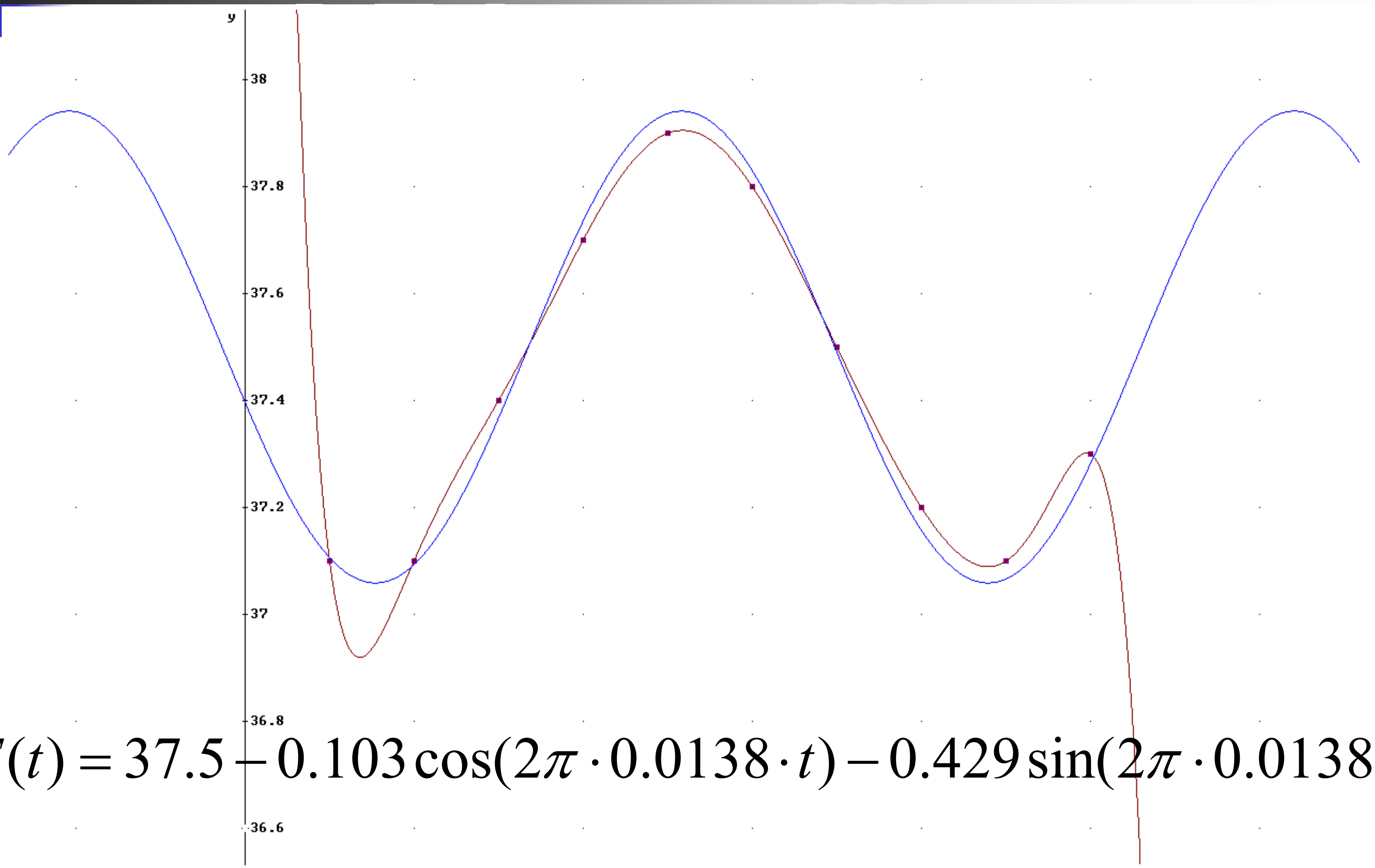
- A polynomial that passes through all the given data points.
  - Given n data points, what would be the degree of the interpolating polynomial?
  - Set up a system of n equations where each data point is substituted into the required n-1 degree equation.
  - Use Derive or a graphic calculator to find the interpolating polynomial

Interpolating Polynomial



$$T(x) = -7.72\times10^{-15}x^9 + 3.97\times10^{-12}x^8 - 8.76\times10^{-10}x^7 + 1.08\times10^{-7}x^6$$
$$-8.19\times10^{-6}x^5 + 3.91\times10^{-4}x^4 - 0.01x^3 + 0.21x^2 - 2.05x + 45$$

Trigonometric Model



$$T(t) = 37.5 - 0.103\cos(2\pi \cdot 0.0138 \cdot t) - 0.429\sin(2\pi \cdot 0.0138 \cdot t)$$

Interpolating Polynomial

- Warning
- Interpolating polynomials should only be used for interpolation
- Even for interpolation these polynomials sometimes oscillate wildly and are not useful