Polynomial Functions

- Section 3.1

- Polynomial comes from the Greek *polus* "many" and the Latin *nomen* "name". So a polynomial function consists of a polynomial expression involving many terms.

Model: U.S. Payments

- The U.S. annual payments to the world in billions of dollars from 1996 to 2001 is given in the table.
  - Determine a model for the data.
  - Find the maximum or minimum payment the model predicts the U.S. will pay.

Year	Pay
1996	227.5
1997	274.2
1998	289.6
1999	294.1
2000	360
2001	295

Model: U.S. Payments

- Scale the data so 1996 corresponds to year 6.
- Use Derive or graphing calculator to determine if the best fit curve is a line or a quadratic.

Year	Pay
6	227.5
7	274.2
8	289.6
9	294.1
10	360
11	295

Model: U.S. Payments

- Linear Model and Error:

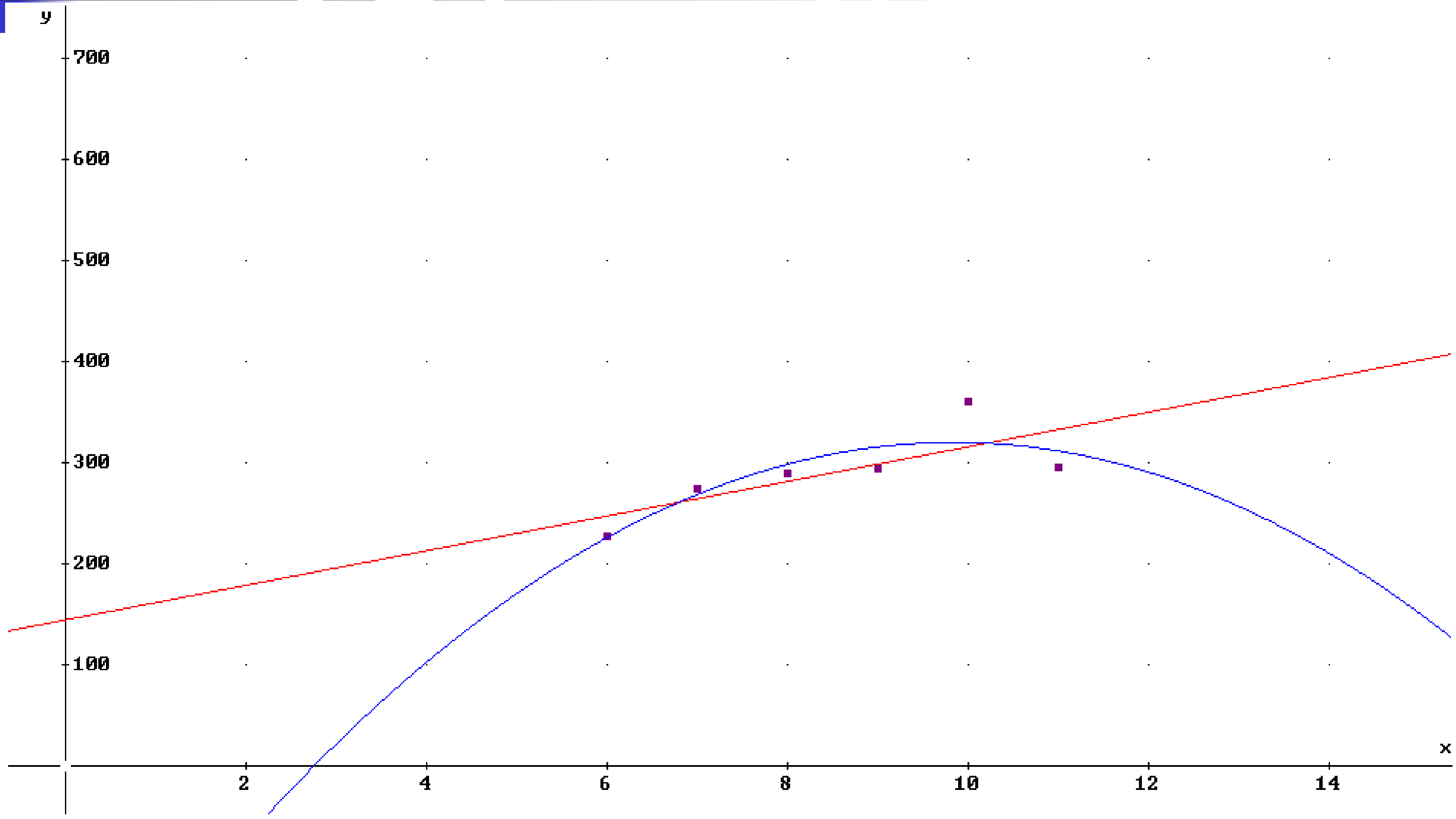
$$p(t) = 17.1t + 144.5 \quad \sigma = 25.7$$

- Quadratic Model and Error:

$$p(t) = -6.4t^2 + 125.3t - 296.9 \quad \sigma = 20.2$$

- Which model is better? Why?

Model: U.S. Payments



Optimization Problems

- We selected the quadratic model because it significantly reduced error and had a leading coefficient of reasonable size.

$$p(t) = -6.4t^2 + 125.3t - 296.9 \quad \sigma = 20.2$$

  - We want to determine when the payment is a minimum or maximum. How does this differ from the problems we have done up to this time?
  - Can we optimize any model using only algebra?

Polynomial Function

- A function whose rule is a polynomial expression.

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

  - Are linear and quadratic functions also polynomial functions?
  - Which power functions $p(x) = A \cdot x^n$ are polynomial functions and which are not?
  - Can we optimize any polynomial function?

Optimizing Quadratic Polynomial Functions

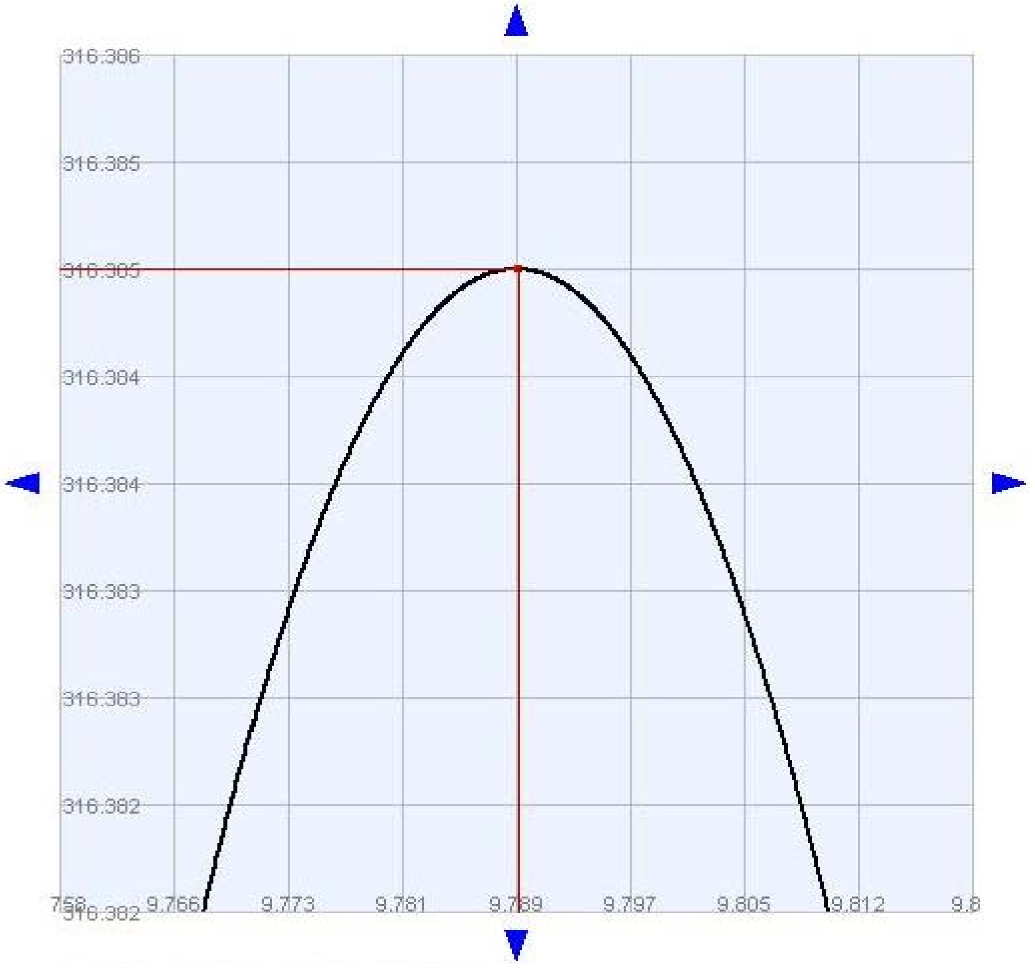
- What is it about quadratic functions that allows us to easily determine a maximum or a minimum value?

  - Solution: Graph is a parabola which has a single largest or smallest value at a point we call the vertex.

Optimizing: Graphic Method

- Create a related function and graph it
- Use the Trace Function to estimate the maximum/minimum value
- Zoom-in on the trace point to determine the vertex within an error of 0.01
- If the graph gets too flat when zooming in then zoom in on the vertical axis only.
  - Solution: (9.79, 316.39) is the vertex to within an error of 0.01. Maximum payment occurs in 1999 and is approximately $316.39 billion.

Optimizing: Graphic Method



Optimizing: Graphic Method

- Advantages
  - Works for any function, not just quadratic functions
  - Requires no calculation
- Disadvantages
  - Must have a general idea of where to look for the maximum/minumum
  - Gives an approximate solution
  - Requires skill in zooming-in

Optimizing: Algebraic Method

- Given a quadratic function model

$$p(t) = -6.4t^2 + 125.3t - 296.9$$

- How can we determine if the optimal value is a maximum or a minimum?
  - Solution: For $y = ax^2 + bx + c$

    If $a > 0$, the parabola opens up and we have a minimum optimal value

    If $a < 0$, the parabola opens down and we have a maximum optimal value

Optimizing: Algebraic Method

- The optimal value occurs at the vertex.
- Find the vertex by completing the square to convert the quadratic function into Standard Form.

  General Form: $f(x) = ax^2 + bx + c$

  Standard Form: $f(x) = a(x-h)^2 + k$

  - Vertex: (h, k) Axis of Symmetry $x = h$

General Formula for Vertex

- Complete square on general quadratic function to get formula for vertex

$$f(x) = ax^2 + bx + c$$

$$f(x) = a\left(x^2 + \frac{b}{a}x\right) + c$$

$$f(x) = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - \frac{b^2}{4a}$$

General Formula for Vertex

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

So the vertex formula is

$$Vertex: \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$$

Example

- Participation Exercise:
- Find the vertex, axis of symmetry and direction for the following

$$f(x) = 2x^2 + 2x + 3$$

  - Solution: Vertex (-0.5, 2.5)
    Axis of Symmetry x = -0.5
    Direction: Up so minimum value