



Inequalities

- Section 2.6
- Inequality – mathematical sentence where the expressions are related by less than $<$, greater than $>$, \leq or \geq .



Model:

Target Heart Rate

- A person's recommended heart rate while exercising can be expressed as a function of age. The base heart rate for a 20 year old is 187 beats per minute. The recommended decrease per year in heart rate is -0.85 .
 - Determine a model for the recommended heart rate r as a function of age in years a .
 - Solution: $r(a) = -0.85a + 187$



Model:

Target Heart Rate

- Use the model $r(a) = -0.85a + 187$ to determine for what ages the recommended heart rate is less than 153 beats per minute.
 - Requires solving the inequality
$$-0.85a + 187 < 153$$



Inequality Properties

- Solve linear inequalities by applying properties of inequalities to isolate the variable.
- Addition Property: $a < b$ then $a+c < b+c$
- Multiplication Property:
 - $a < b$ and $c > 0$ then $ac < bc$
 - $a < b$ and $c < 0$ then $ac > bc$
- Transitive Property: $a < b$, $b < c$, then $a < c$
 - Use these properties to solve the linear inequality
$$-0.85a + 187 < 153$$



Linear Inequality Types

- Try solving the following inequalities

$$4x + 8 \geq x - 1$$

$$-2 < -2x + 6 < 4$$



Quadratic Inequalities

- The approximate number of accidents in one month involving drivers between 16 and 30 years of age inclusive is modeled by the quadratic of best fit

$$n(a) = 2a^2 - 80a + 5000$$

- Determine for what years of age the number of accidents exceeds $n > 4,300$



Numeric Method

- Find a related function for the inequality
 - Set inequality to zero
 - Replace 0 with y
 - Generate table using related function
$$2a^2 - 80a + 5000 > 4300$$
 - Related function: $y = 2a^2 - 80a + 700$
 - Use Derive or graphing calculator to create tables and zoom-in to solve



Numeric Method

- Generate table using related function

$$y = 2a^2 - 80a + 700$$

- When is $n > 4,300$ to within an error of 0.1?

a	y	a	y
16	-68	27	-2
17	-82	27.1	0.82
18	-92	27.2	3.68
19	-98	27.3	6.58
20	-100	27.4	9.52
21	-98	27.5	12.5
22	-92	27.6	15.52
23	-82	27.7	18.58
24	-68	27.8	21.68
25	-50	27.9	24.82
26	-28	28	28
27	-2		
28	28		
29	62		
30	100		

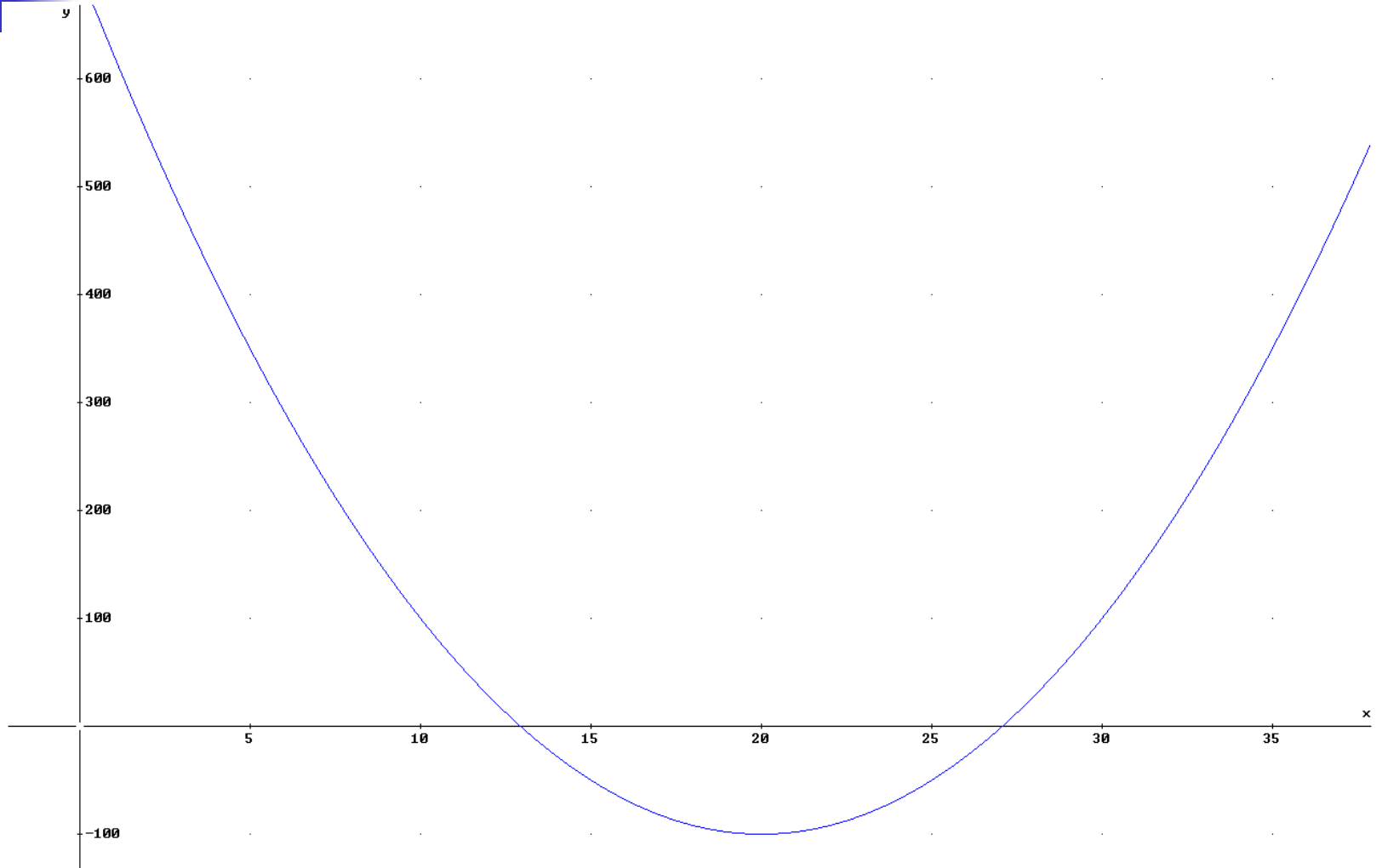


Graphic Method

- Duplicate the numeric method with a graph
 - Graph related function
$$y = 2a^2 - 80a + 700$$
 - Use Derive or a graphing calculator to create graph and zoom-in to solve
 - Examine the graph on the next slide and discuss why one of the solutions should be omitted.

Graphic Method

$$y = 2a^2 - 80a + 700$$





Algebraic Method

- Sign Analysis
 - Set the inequality to zero
 - Factor the expression
 - Plot the zeros on a number line
 - Determine over what interval the expression is positive or negative
 - If expression > 0 take positive intervals
 - If expression < 0 take negative intervals



Algebraic Method

- Use sign analysis to solve the following inequalities
 - $-x^2 - 40 > -13x$
 - $3x^2 - 2x \geq 5$



Algebraic Method

- Use sign analysis to solve the Accident Problem inequality
 - $2a^2 - 80a + 700 > 0$
- What do we do when the quadratic does not factor easily?
 - Apply the Quadratic Formula

$$a = 20 \pm 5\sqrt{2}$$

$$a \approx 12.9 \text{ or } 27.1$$

Algebraic Method

- Use zeros to complete factorization
 - $2a^2 - 80a + 700 > 0$
 - $(a - 12.9)(a - 27.1) > 0$
- Apply Sign Analysis

