



Quadratic Equations

- Section 2.5
- Methods of Solving Quadratic Equations
 - Square Root Method
 - Completing the Square
 - Quadratic Formula



Model: Higher Ed. Cost

Find a model for the average cost of tuition and fees per semester for public 4-year colleges in the U.S.

When will the cost reach \$3500?

Year	Cost
1975	599
1980	840
1985	1386
1988	1726
1989	1846
1990	2006

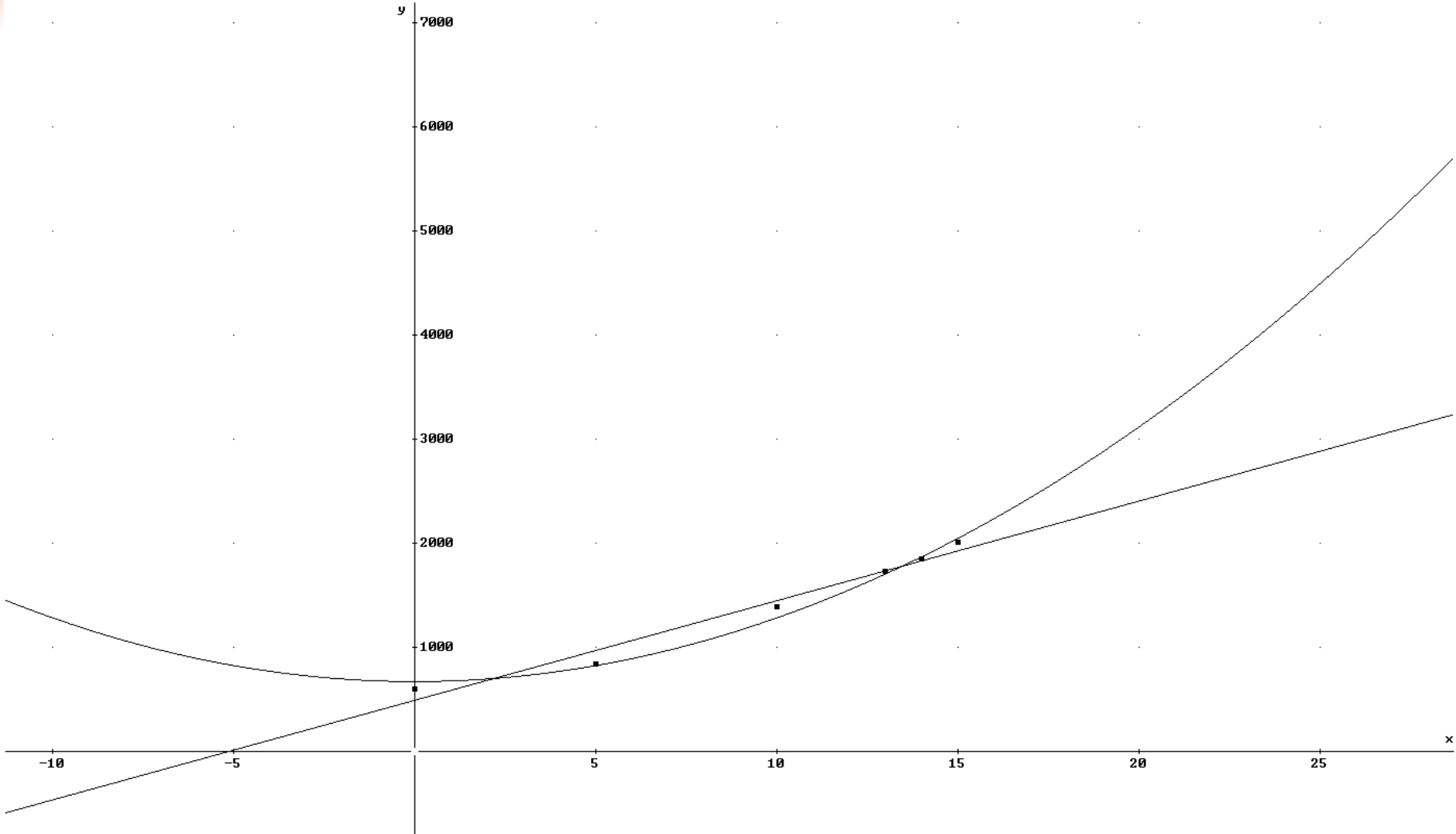


Model: Higher Ed. Cost

- Is the data linear or curvilinear?
- Ladder of Powers
 - Examine power functions of form $y = ax^n + b$
 - Derive or Grapher to Fit
 - $y = 95.7x + 491.4$ (sd 80.5)
 - $y = 6.1x^2 + 671.7$ (sd 56.0)

Year	Cost
1975	599
1980	840
1985	1386
1988	1726
1989	1846
1990	2006

Scatter plot & Models





Square Root Method

- Let $y = 3500$ in model

$$y = 6.1x^2 + 671.7$$

- Resulting quadratic equation has no linear term

$$6.1x^2 + 671.7 = 3500$$

- How can we parallel the method of solving linear equations to solve this quadratic equation?



Square Root Method

- Solving a quadratic with no linear term
 - Isolate the square term

$$6.1x^2 + 671.7 = 3500$$

$$6.1x^2 + 671.7 - 671.7 = 3500 - 671.7$$

$$6.1x^2 = 2828.3$$

$$x^2 = 2828.3/6.1$$

- Square root to find x

$$x = \pm \sqrt{\frac{2828.3}{6.1}} \approx \pm 21.5$$



Solving Quadratic Equation with a Linear Term

- Quadratic of Best Fit with Linear Term
$$y = 3.7x^2 + 38.9x + 587.6 \quad (\text{sd } 22.4)$$
- Let $y = 3500$ and solve resulting quadratic equation with a linear term
$$3.7x^2 + 38.9x + 587.6 = 3500$$
- How do we solve such equations?



Completing the Square Method

- Solve by converting to a perfect square and using the Square Root Method

$$x^2 + 4x - 5 = 0$$

Isolate the x terms

$$x^2 + 4x = 5$$

Complete the square

$$x^2 + 4x + 2^2 = 5 + 2^2$$

$$(x+2)^2 = 9$$



Completing the Square Method

- Square Root and solve

$$(x+2)^2 = 9$$

$$\sqrt{(x+2)^2} = \pm\sqrt{9}$$

$$x + 2 = 3 \quad \text{or} \quad x + 2 = -3$$

$$x = 1 \quad \text{or} \quad x = -5$$



Quadratic Formula

- Complete the square on the general quadratic to get a general solution

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$



Quadratic Formula

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Quadratic Formula

- Use the Quadratic Formula to solve $3.7x^2 + 38.9x + 587.6 = 3500$

$$3.7x^2 + 38.9x - 2912.4 = 0$$

So $a = 3.7$, $b = 38.9$, and $c = -2912.4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x \approx -33.7 \quad \text{or} \quad x \approx 23.3$$



The Discriminant

- Using the Quadratic Formula, how can we determine if the solutions are real or complex?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Discriminant: $b^2 - 4ac$
 - Determine what type of solutions $3x^2 + 5x - 7 = 0$
 - Solution: $5^2 - 4(3)(-7) > 0$ so there are two real solutions



Numeric Method

- What is the related function for the Higher Education problem?

$$3.7x^2 + 38.9x + 587.6 = 3500$$

$$f(x) = 3.7x^2 + 38.9x - 2914.4$$

- Use Derive to generate a table and zoom-in to find an error less than 0.01



Graphic Method

- Graph the related function for the Higher Education problem.

$$f(x) = 3.7x^2 + 38.9x - 2914.4$$

- Use Derive or the Grapher to generate a graph and zoom-in to find an error less than 0.01