Linear Equations

- Section 2.4

  - Equation comes from the Latin *aequus* meaning "even" or "level". An equation is a statement in which two algebraic expressions are level or equal.
  - Equal symbol = comes from two parallel lines, because nothing is more equal then parallel lines (Robert Recorde 16th Century)

Model: Sociology

- The percentage of the total population in college was 15% in 1940 and 27.5% in 1950. Since 1940 the percentage in college has increased at a study rate of 1.25% per year.
- In 1980 what was the percent of the population in college?
- In what year was the percent of the population in college 33%?

Decal Heuristic to Find Model

- Describe the Problem
- What is the setting?
  - Solution: General rate problem

    Total = rate x time + base amount
- What is the question?
  - Solution: Unknowns are year t and percent attending college p.
  - Find a model for p in terms of t so we can find the percent of the population in college in 1980

Decal Heuristic to Find Model

- Describe the Problem
- What are the Facts?
  - Solution: p = 15 in 1940.
  - Let 1940 be t = 0, then the base amount is p = 15.
  - Rate of increase is 1.25 per year
- What are the Distractors?
  - Do not need the information about 27.5% college attending in 1950.

Decal Heuristic to Find Model

- Explore: Create a model using the facts and the known relationship
- Create Model:
  - Model: $p(t) = 1.25\,t + 15$
- Apply the Model: What is p when t is the year 1980?
  - Substitute $t = 1980 - 1940 = 40$ into the model for the independent variable t

Substitution for Independent Variable

- When substituting for the independent variable in a model, all we need is arithmetic to find the answer.

  - P(40) = 1.25 (40) + 15

    P(40) = 65 percent in college

Substitution for Dependent Variable

- When substituting for the dependent variable in a model, we need methods for solving a linear equation.
  - $33 = 1.25\ t + 15$

Algebraic Method of Solving a Linear Equation

- Use the properties of equality (pg 302) to isolate the variable.

$$33 = 1.25\,t + 15$$

$$33 - 15 = 1.25\,t + 15 - 15$$

$$18 = 1.25\,t$$

$$18/1.25 = 1.25/1.25\,t$$

$$t = 14.4 \text{ years after } 1940$$

Algebraic Method of Solving a Linear Equation

- Use the Algebraic Method to solve the following equations
  - 7x+3 = 8
  - Multiple occurrence of variable
  - 3(4x - 2) + 7 = 5(x+1)
  - Non-linear reducible to linear form

$$\frac{3}{2x-1}+2=\frac{3x}{2x-1}$$

Numeric Method of Solving a Linear Equation

- Create a Related Function for the equation
  - Set the equation equal to zero

$$33 = 1.25\,t + 15$$

$$1.25\,t - 18 = 0$$

  - Substitute for zero with y to get a function of two variables

$$y = 1.25\,t - 18$$

Numeric Method of Solving a Linear Equation

- Create a Table using the related function

$$y = 1.25\,t - 18$$

- Over what interval does the related function cross the x-axis?
- How does this relate to the original problem?
  - Solution: The y values in the table change signs on (10,20) so there is a solution on this interval

t	y
0	-18
10	-5.5
20	7
30	19.5
40	32
50	44.5
60	57
70	69.5
80	82
90	94.5
100	107

Numeric Method of Solving a Linear Equation

- Estimate solution as midpoint of interval t = 15.
- Maximum error in approximation is no more than the interval width of 10.
- Zoom-in on solution interval to get a better approximation

t	y
0	-18
10	-5.5
20	7
30	19.5
40	32
50	44.5
60	57
70	69.5
80	82
90	94.5
100	107

Zoom-in on Table

- Zoom-in by scale of 10 until you reach an error of no more than 0.01.
- Use Derive or Graphing Calculator to make tables.
  - Solution: Solution is on interval (14, 15). Estimate as 14.5 with error of no more than 1

t	y
10	-5.5
11	-4.25
12	-3
13	-1.75
14	-0.5
15	0.75
16	2
17	3.25
18	4.5
19	5.75
20	7

Graphic Method of Solving a Linear Equation

- Parallels the numeric method used with tables.
- Plot the related equation $y = 1.25x - 18$.
- Approximate the x-intercept from the graph.
- Zoom-in to get more accurate solutions – error is no more than the x-axis scale of the graph
- Use technology to approximate the solution for the College Problem.

Graphic Method of Solving a Linear Equation

- What is an approximation of the year using the graph?
  - Solution: t=14
- What is the maximum error in this approximation?
  - Error is no more than 4 since the x-scale is 4