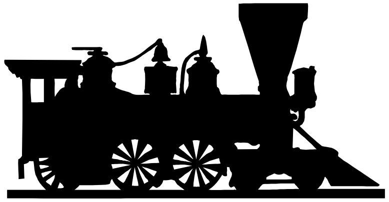
Mathematical Modeling – Known Relationships

- Section 2.1
  - Model: a scaled down or simplified version of a complex situation that allows one to answer important questions.
  - Everyday examples of model
    Model Train or Road Map



Model: Septic Tank

A septic tank is the shape of a cylinder with a length of 6 feet, with a ½ sphere on each end with a diameter of d feet. The tank must be buried 3 feet under the ground. Determine the volume of the tank when the diameter is 4 feet?

Real World Applications

- Two types of problems
  - Problems modeled by a **previously known relationship** (Section 2.1).
  - Problems requiring **data analysis** since the relationship underlying the model is unknown (Section 2.2 & 2.3).

Properties of a good model

- Simplifies the phenomenon it represents
- Allows for interpolation and extrapolation from the known values or relationships.
- In Algebra the models are functions.

DECAL Heuristic

- A heuristic is an organized method of solving problems.
- Stimulates reasoning leading to a plausible solution.
- Use DECAL Heuristic to solve the Septic Tank Problem.

Step 1: Describe the Problem

- Setting: Identify known relationship or law that underlies the problem.
  - Commonly Known Formulas: Page 231
  - Solution: The setting is not a sewer problem, that is a context.
  - The underlying math relationship is finding the volume of a solid.

Volume of Cylinder: V = Base area X Height

Volume of Sphere:

$$V = \frac{4}{3}\pi r^3$$

Step 1: Describe the Problem

- Question: Identify the unknowns and label them as variable quantities. Clearly identify the question you are answering.
  - Solution: Question is to find the volume V of the tank.
  - Variables: V = volume of tank

    l = length of tank

    r = radius of tank

Step 1: Describe the Problem

- Facts: List and identify all key facts.

  - Solution: l = 6 feet
    Find volume when d = 4 feet

Step 1: Describe the Problem

- Distractors: List all extraneous information – information not needed to solve the problem.

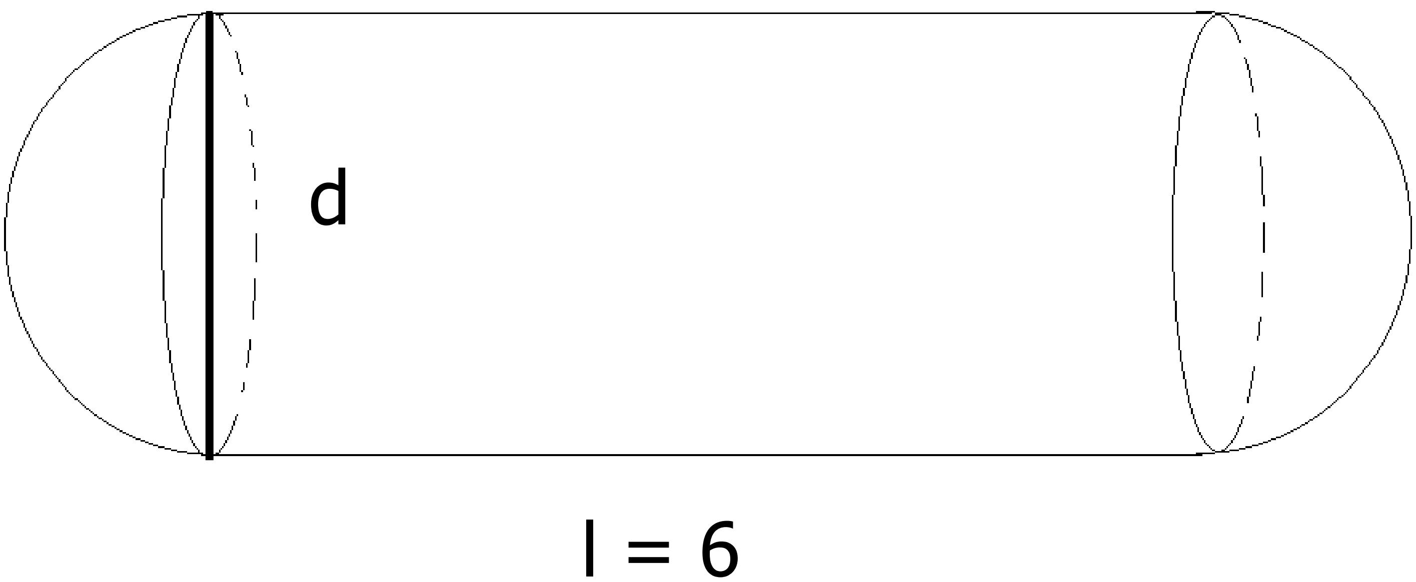
  - Solution: Burying tank 3 feet.

Step 2: Explore

- Critical thinking part of the heuristic. Determine how to move from Describing the problem to formulating a function model.
- Employ variety of strategies (Pg. 232)
  - Deductive Reasoning
  - Inductive Reasoning
  - Analytic Reasoning
  - Recursive Reasoning
  - Visualizing

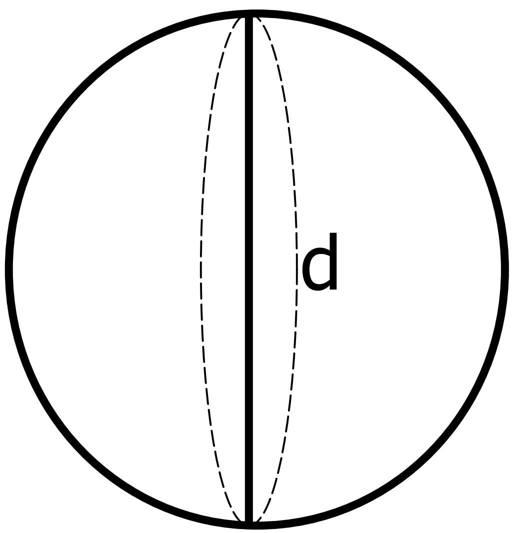
Step 2: Explore

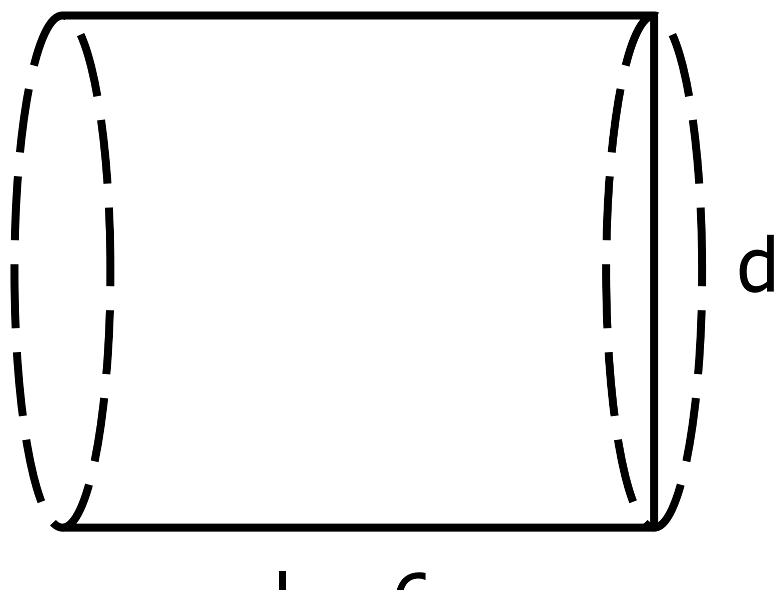
Visualize the problem by drawing a sketch and labeling the parts.



Step 2: Explore

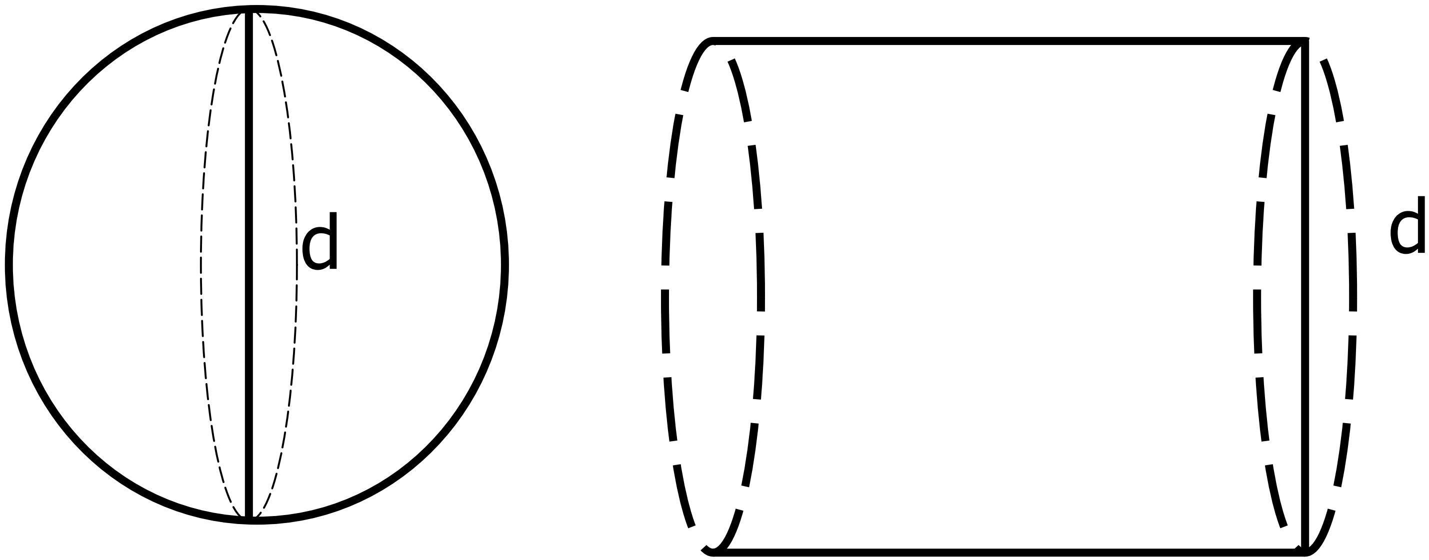
Analytic Reasoning: break the problem into parts.





Step 2: Explore

- Deductive Reasoning: apply known relationships
  - Total Vol. = Sphere Vol. + Cylinder Vol.
  - Total Vol. = $\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 + 6\pi\left(\frac{d}{2}\right)^2$



Step 3: Create Model

Total Vol. = Sphere Vol. + Cylinder Vol.

- Total Vol. = $\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 + 6\pi\left(\frac{d}{2}\right)^2$
- Function Model

$$V(d) = \frac{\pi}{6}d^3 + \frac{3\pi}{2}d^2$$

Step 4: Apply the Model

- Let d = 4

$$V(d)=\frac{\pi}{6}d^3+\frac{3\pi}{2}d^2$$

$$V(4)=\frac{\pi}{6}4^3+\frac{3\pi}{2}4^2$$

$$V(4)=\frac{104\pi}{3}\approx 108.9\ ft^3$$

Step 5: Link to New Situations

- Review the Problem

$$V(4) = \frac{104\pi}{3} \approx 108.9 \text{ Cubic Feet}$$

which seems to be a reasonable answer

Step 5: Link to New Situations

- Extend the Problem: What would be the capacity of a tank with a diameter of d = 10 feet?

$$V(d) = \frac{\pi}{6}d^3 + \frac{3\pi}{2}d^2$$

$$V(10) = \frac{\pi}{6}10^3 + \frac{3\pi}{2}10^2$$

$$V(10) = \frac{950\pi}{3} \approx 994.8$$