Function Inverse

- Section 1.5

  - Inverse comes from the Latin *in vertere* meaning “to turn inside out”. In mathematics a functions inverse turns the original function inside out, giving back whatever value has been substituted into it.

Model: Avoidance

- People often avoid certain situations, such as public speaking. In an experiment on avoidance, rats received mild shocks or food from a goal box. Avoidance was measured in pull (in grams) away from the shock box when the rat is placed d centimeters from it.

Avoidance Data

- Data is given as (Distance, Pull)
- **Property of Inverse:** Inverse function reverses the original function If (x,y) is on the original function, then (y,x) is on the inverse function.
- What is the inverse point for (5,223.3)?
  - Solution: (223.3, 5)

Distance	Pull
5	223.3
10	216.7
15	210
20	203.3
25	196.8
30	190.2
35	183.4
40	176.8
45	170
50	163.2
55	156.6
60	150
65	143.3
70	136.7
75	130
80	123.3
85	116.9
90	110
95	103.3
100	96.7

Inverse Concept

- Additive Inverse – Given 5, what is the additive inverse?
  - Solution: -5
- When you add the inverse to 5 what do you get?
  - Solution: The additive identity 0
  - (number)+(add. inverse)=additive identity

Inverse Concept

- Multiplicative Inverse – Given 5, what is the multiplicative inverse?
  - Solution: 1/5
- When you multiple the inverse by 5 what do you get?
  - Solution: The multiplicative identity 1
  - (number) (mult. inverse)=mult. identity

Function Inverse f^{-1}

What is the operation when taking function inverses?

- Solution: Composition of functions

- What is the function identity?

Let f(x)= 2x-1, let i(x) be the identify function. Then by definition of identity

$$(f \circ i)(x) = f(x)$$

Function Identity i(x)

$$(f \circ i)(x) = f(x)$$

$$f(i(x)) = f(x)$$

$$2 \cdot i(x) - 1 = 2x - 1$$

- What must i(x) equal for this statement to be true?
  - Solution: The identity function is

i(x) = x

Inverse Function

f^{-1} is the inverse function of *f* if and only if

$(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$

Where the domain of f equals the range of the inverse function

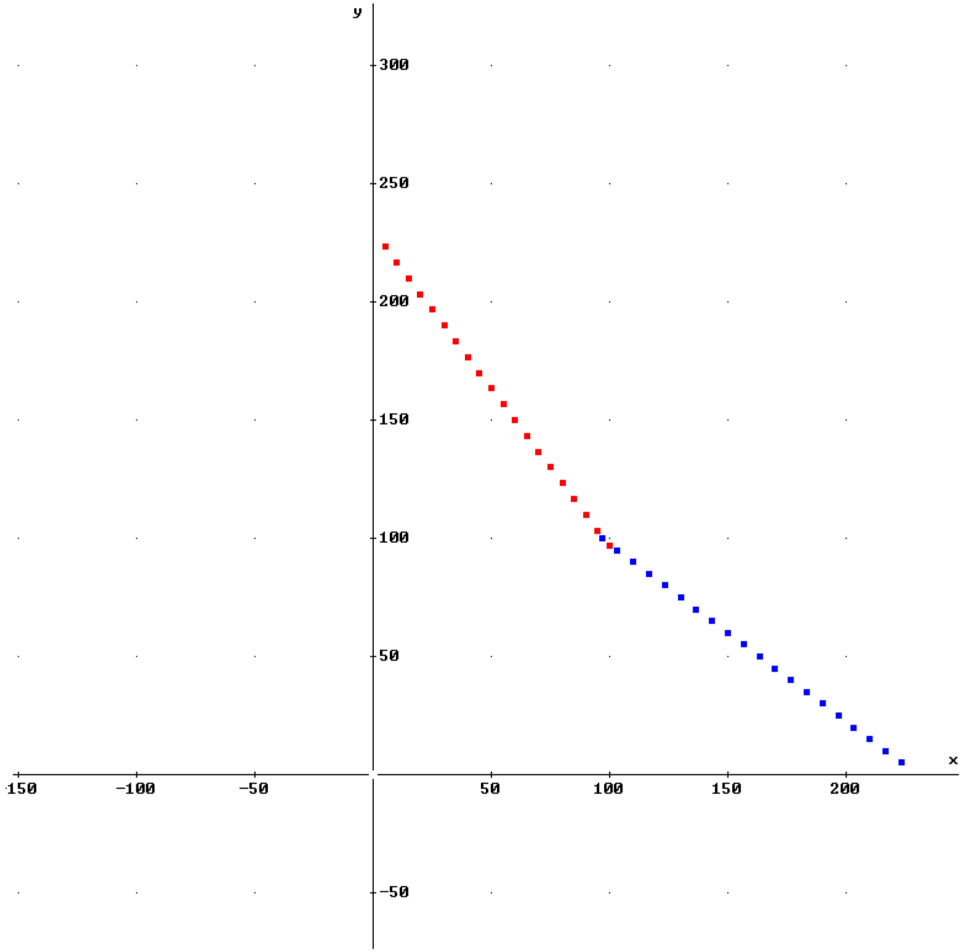
and the range of f equals the domain of the inverse function.

Inverse of Graph

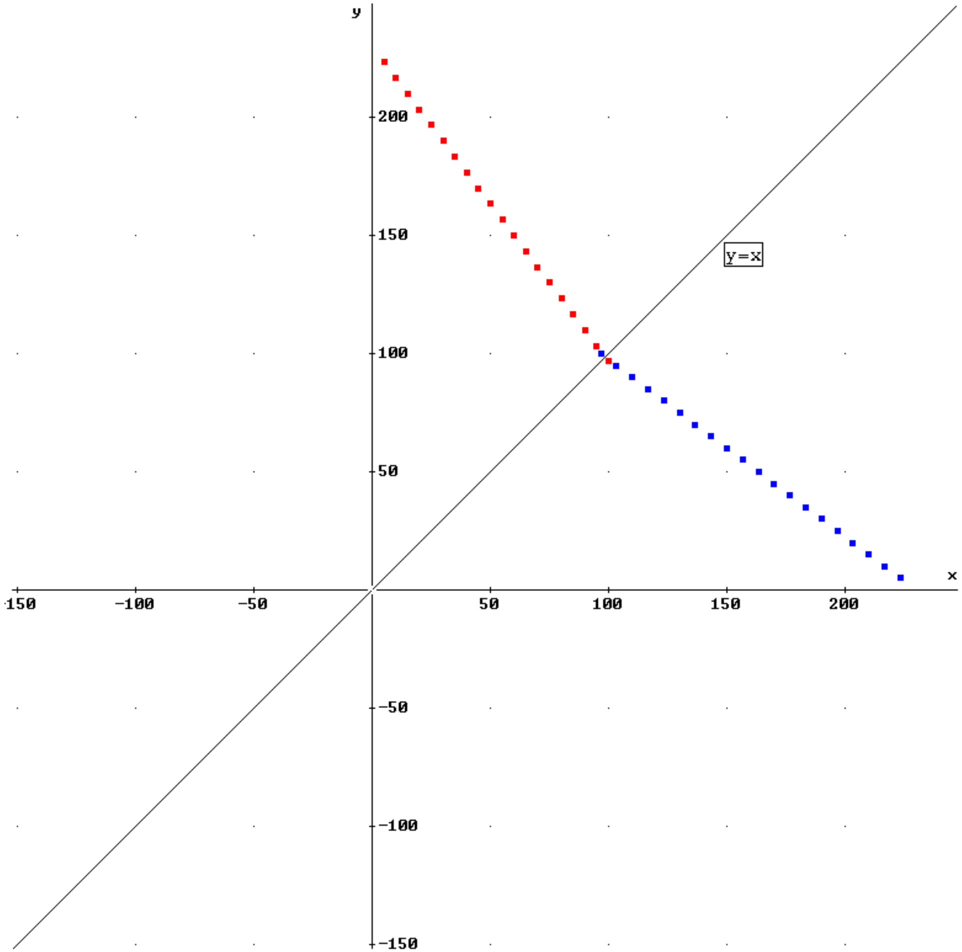
- Given if (x,y) is a point on a function, (y,x) is a point on the inverse function – What is the relationship between the graph of a function and its inverse?
- Examine the graph of the avoidance data and the inverse of the avoidance data to answer this question.

Avoidance Data (red)

Inverse Data (blue)



Avoidance Data and Inverse Data are symmetric about the line y = x.



Inverse of an Equation

- The best fit model for the avoidance data is

$$p(d) = -\frac{4}{3}d + 230$$

- We want to find the inverse of this function so that given a pull *p* we can determine the distance *d*. How can we find the inverse function *d(p)* ?

Inverse of an Equation

- Replace the function notation p(d) with y and d with x to make the manipulations easier.

$$y = -\frac{4}{3}x + 230$$

- If (x,y) is the original function, then (y,x) is on the inverse function. So switch the dependent and independent variables to get the inverse function.

$$x = -\frac{4}{3}y + 230$$

Inverse of an Equation

- Solve for y to get the equation as a function of x.

$$x = -\frac{4}{3}y + 230$$

$$x - 230 = -\frac{4}{3}y + 230 - 230$$

$$-\frac{3}{4}(x - 230) = y$$

$$y = -\frac{3}{4}x + \frac{345}{2}$$

Inverse - Avoidance Function

- Now substitute the original variables back into the inverse equation – y is d(p) and x is p.

$$y = -\frac{3}{4}x + \frac{345}{2}$$

$$d(p) = -\frac{3}{4}p + \frac{345}{2}$$

One-to-one Function

- A function has an inverse function only if the original function is one-to-one.
- A function is one-to-one if when

  f(a) = f(b), then a = b.
- Compare this definition to that of being a function. What is a geometric test to determine if a function is one-to-one?
  - Solution: Horizontal Line Test.

Is this function one-to-one?