Function – Process & Object

- Section 1.4
- Operations on functions: +, -, •, / and composition
  - Composition of functions comes from the Latin *co-* meaning "together with" and *ponere* meaning "to put". So composition of functions is literally putting one function inside another.

Model: Gross National Product

- The gross national product N of the U.S. is determined by taking the gross domestic product D, adding the income receipts R from the rest of the world, and subtracting the income payments P by the U.S. to the rest of the world.

- What is the U.S. Gross National Product?

Year	G.D. Product	Income	Debt
1991	5986.2	167.7	143
1992	6318.9	151.1	127.6
1993	6642.3	154.4	130.1
1994	7054.3	184.3	167.5
1995	7400.5	232.3	211.9
1996	7813.2	245.6	227.5
1997	8318.4	281.3	274.2
1998	8781.5	286.1	289.6
1999	9268.6	313.8	320.5
2000	9872.9	384.2	396.3
2001	10208.1	335.2	340.5

All Values in Billions of Dollars

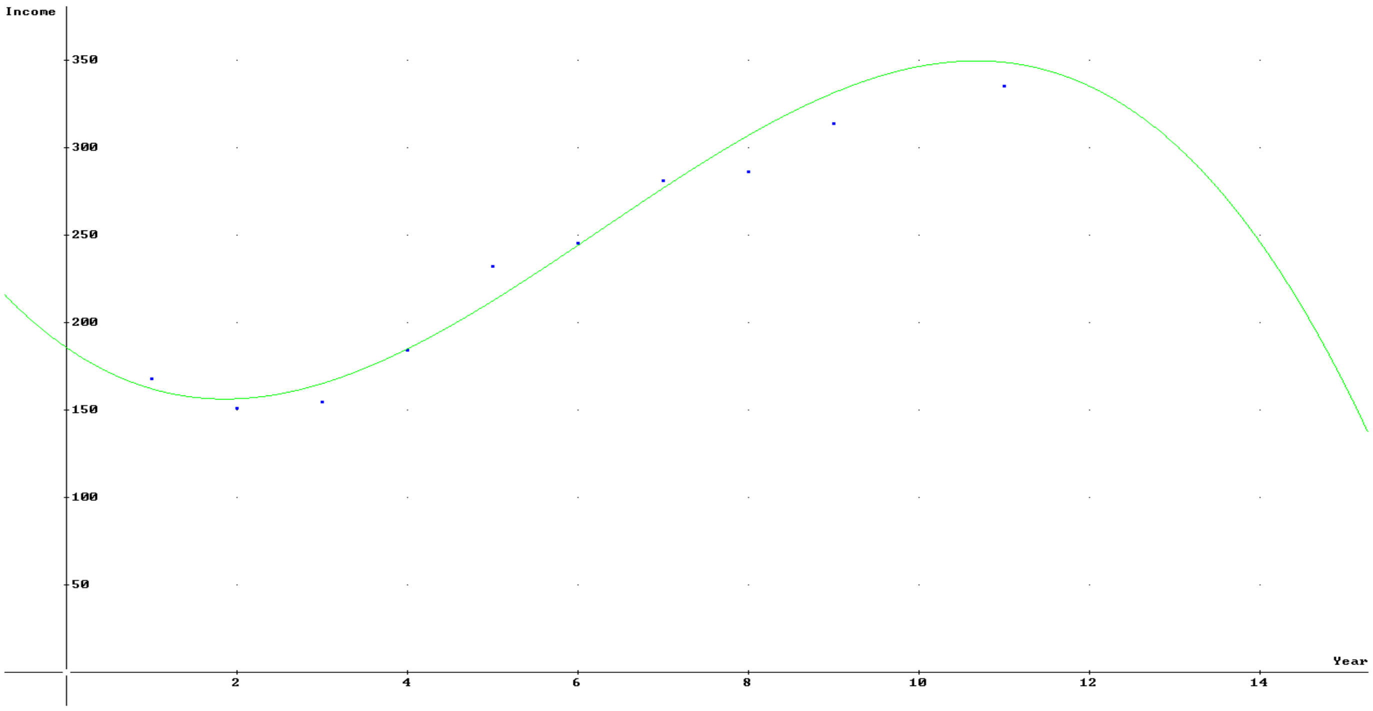
Trend Analysis

- Examine the data to determine trends in U.S. Income. When is income increasing?
- Examining trends requires interpreting the table as a whole entity.
- This is an object conception of function.

Year	Income
1991	167.7
1992	151.1
1993	154.4
1994	184.3
1995	232.3
1996	245.6
1997	281.3
1998	286.1
1999	313.8
2000	384.2
2001	335.2

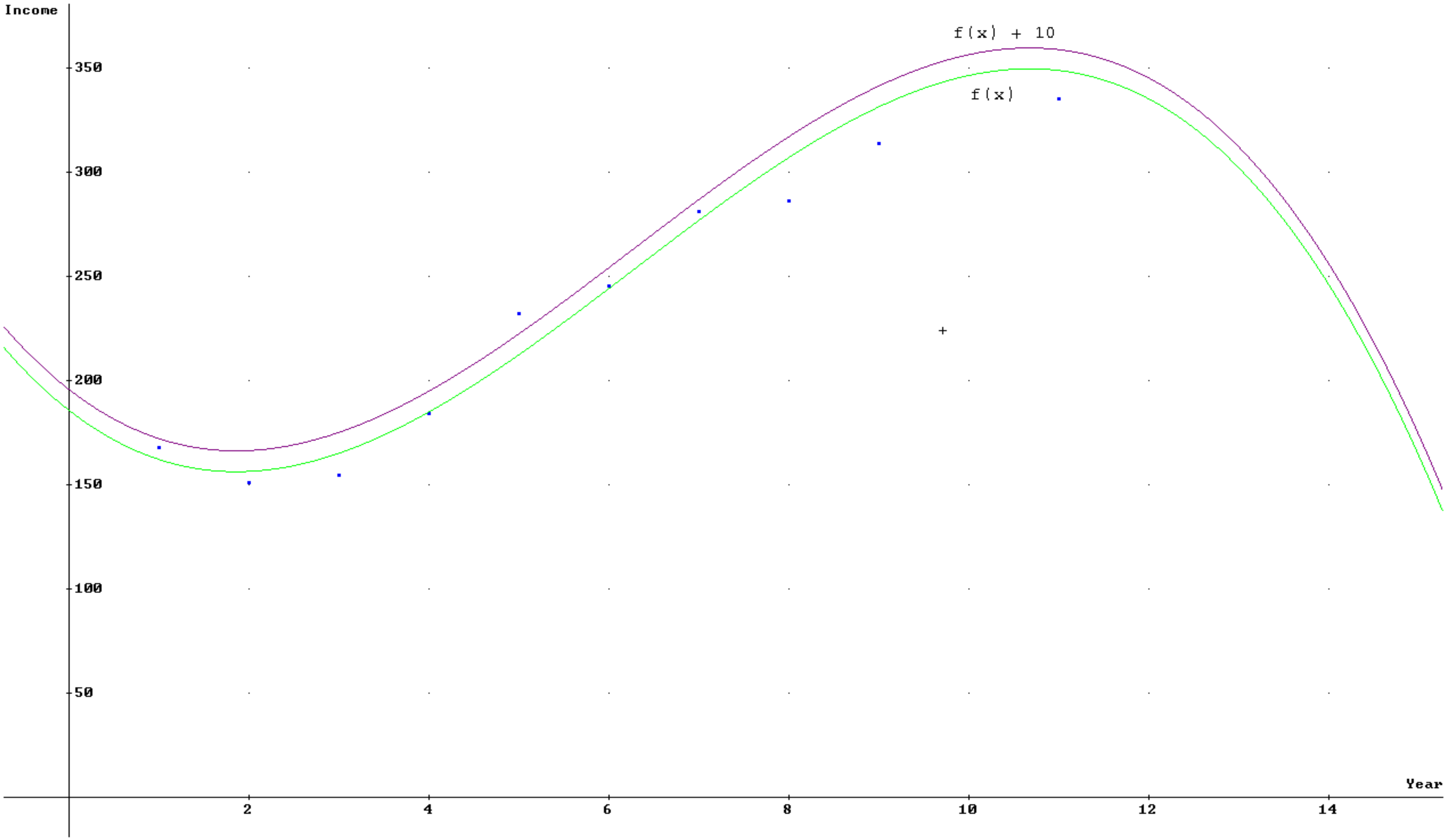
Trend Analysis

- Examine the graph of the data and its best fit curve to determine trends in U.S. Income. When is income increasing?



Transformation of a Function

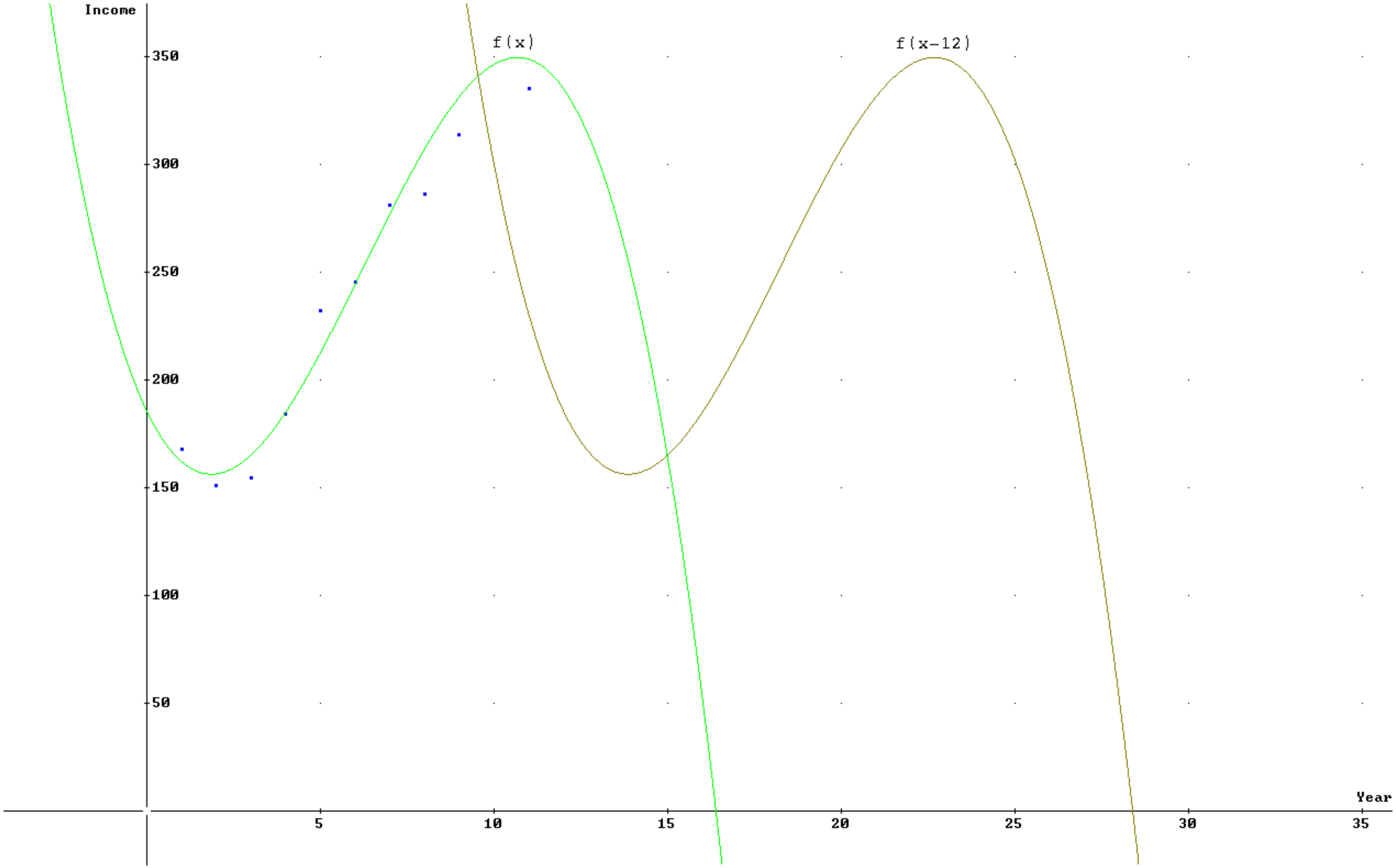
- A vertical or horizontal shift, a stretch or compression, a reflection, or any combination of these of the original graph.
  - Let the graph of U.S. Income be f(x). What is the effect of an increase in income of 10 billion dollars every month on f(x)?



- Solution: f(x) + 10 is a vertical translation up 10.

Transformation of a Function

- Let the graph of U.S. Income be f(x). What is the effect of predicting a repeat in the income cycle over the next 12 years?



- Solution: f(x-12) is a horizontal translation right 12.

Transformation Types

- In Lab 3 we will explore the effect of three types of translations.
- Multiply function F by a real number *a*
  - $G(x) = a \bullet F(x)$
- Add/subtract a real number *b* from the independent variable
  - $G(x) = F(x+b)$
- Add/subtract a real number *c* from the dependent variable
  - $G(x) = F(x)+c$

Algebra of Functions

- Functions can be +, -, •, / using the rules in the Preliminary Chapter.

- Find the net gain from income receipts and income payments for the U.S by examining the table, graph, and equation.

- How do we find the net gain or loss using the table?
  - Solution: subtract element by element
  - Gain = Income - Debt

Year	Income	Debt
1991	167.7	143
1992	151.1	127.6
1993	154.4	130.1
1994	184.3	167.5
1995	232.3	211.9
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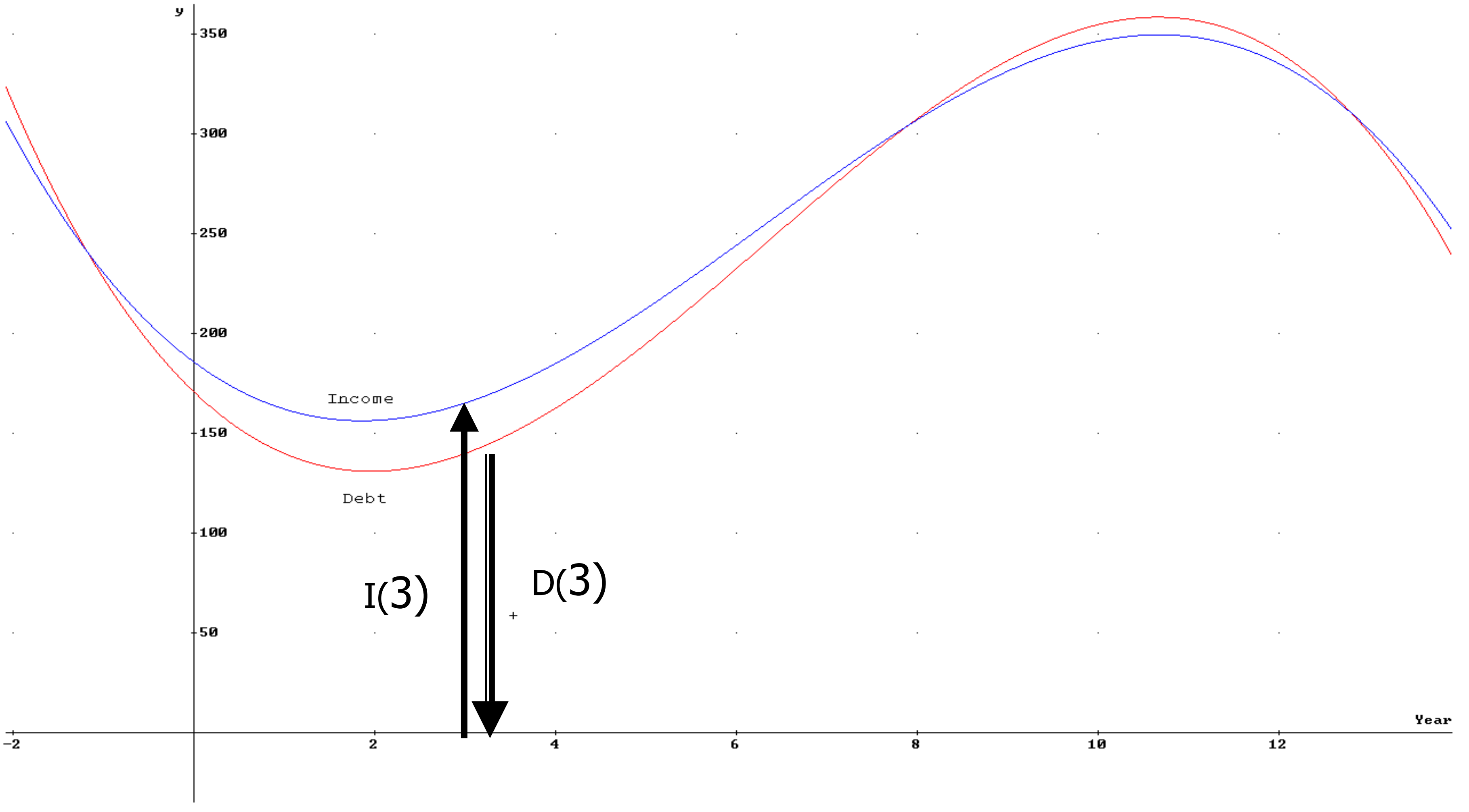
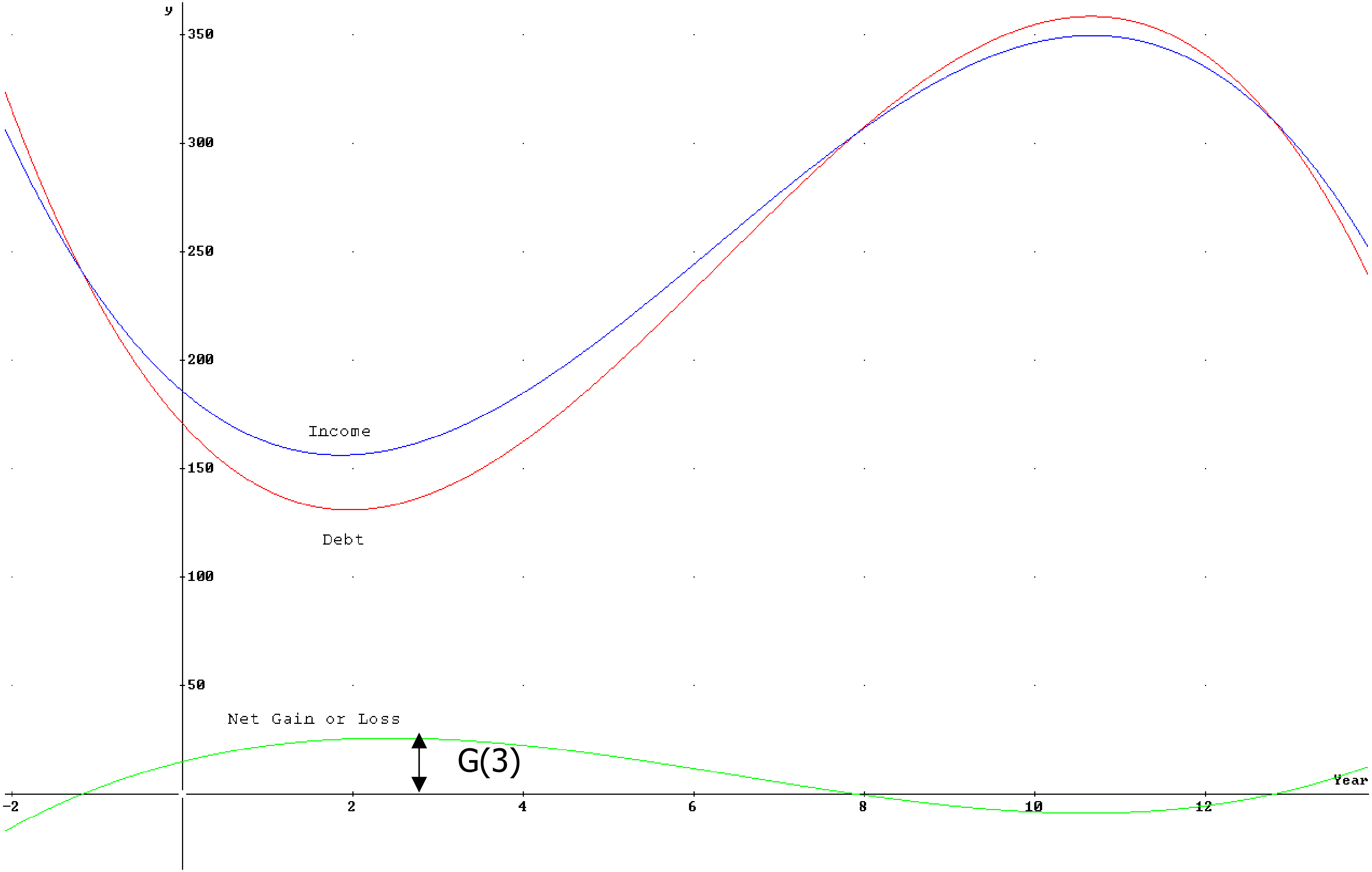


- How do we find the net gain or loss using the graph?
  - Solution: Subtract point by point



- When is there a net gain? Net loss?

Algebra of Functions

- How can we find the Gross National Product using equations for the Gain and the Gross Domestic Product?
  - Best fit curve for GDP

    $$D(x) = 11.1x^2 + 299.1x + 5663.5$$

  - Best fit curve for Gain

    $$G(x) = 0.1x^3 - 2.5x^2 + 9.7x + 14.9$$

  - Solution: Add using the rules for adding polynomial expressions

    $$N(x) = 0.1x^3 + 8.6x^2 + 308.8x + 5678.4$$

Domain of Functions +, -, •, /

- Let the function f have domain *dom f* and g have domain *dom g*, then what are the domains of the sum, difference, product, and quotient of f and g?
  - f+g, f-g, and f•g have domain

    $$\text{dom } f \cap \text{dom } g$$

  - f/g has the same domain with one additional restriction, $g(x) \neq 0$

Composition of Functions

- Composition of function f with function g is performed by letting f act on g as an input.
- Substitute g for x in f.
- Notation:

$$(f \circ g)(x) = f(g(x))$$

Composition Example

- Find $f \circ g$ and determine the domain of the composition when

$$f(x) = \frac{1}{x^2 - 9} \qquad g(x) = \sqrt{x - 1}$$

  - Solution: Substitute g(x) for x in f and simplify

$$(f \circ g)(x) = f(g(x))$$

$$(f \circ g)(x) = f\left(\sqrt{x-1}\right)$$

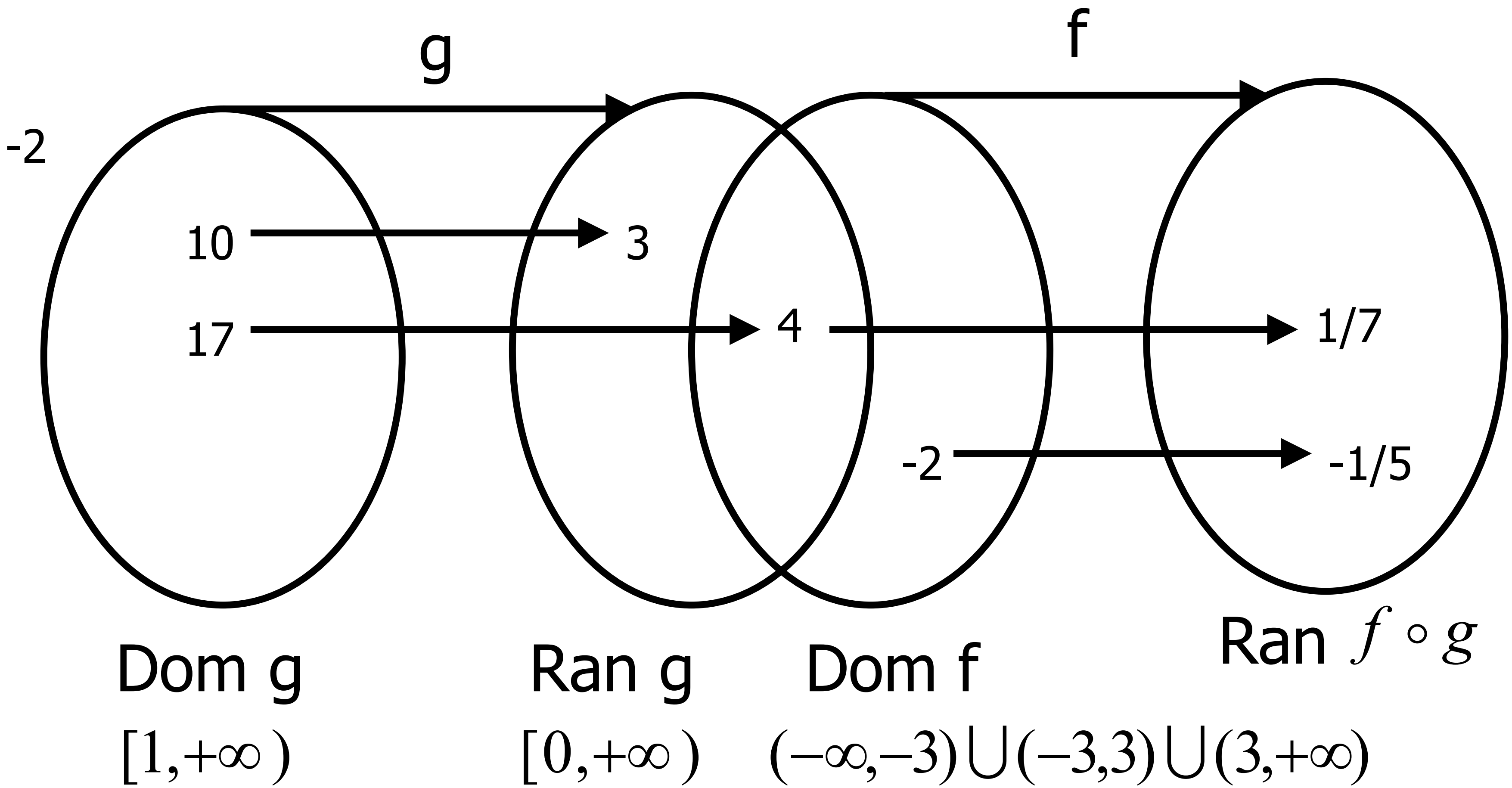
$$(f \circ g)(x) = \frac{1}{\left(\sqrt{x-1}\right)^2 - 9}$$

$$(f \circ g)(x) = \frac{1}{(x-1)-9} = \frac{1}{x-10}$$

Is the domain of the composition all reals except 10?

Composition of Function $f \circ g$

Follow Example 6, Section 1-4



Model - Composition

In biological systems carnivores eat herbivores and herbivores eat plants. In a theoretical system we have Mountaineers eating deer and deer eating grass. Given the following models for consumption, determine the Mountaineers dependence on grass.

$$M(d) = \sqrt{d - 20} \qquad d(g) = g^2 + 80x$$

- Solution: Take the composition M(d(g)) to get M(g)

$$M(g) = \sqrt{g^2 + 80x - 20}$$