



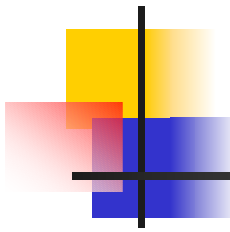
Linear Relationships

- Section 1.1
 - Relation comes from the Latin *re-* “back again” and *latus*, “carried”. Relations are those that you are carried back to along blood lines.



Model: Fuel Economy

- The U.S. Department of Energy claims that every gallon of gas your vehicle burns puts 20 lbs of carbon dioxide into the atmosphere. CO_2 is one of the major greenhouse gasses causing global warming. Let's explore a model comparing data on engine size and fuel economy.



US Department of Energy, 1999 Fuel Economy Guide

Make and model	Engine Size (liters)	Highway Mileage (mpg)
BMW M Coupe	3.2	24
Chevy Corvette	5.7	28
Lamborghini Diablo	5.7	13
Mazda MX-5 Miata	1.8	29
Chevy Cavalier	2.4	33
Ford Mustang	4.6	24
Lexus SC400	4.0	25
Pontiac Firebird/TA	5.7	28
Saturn SC	1.9	38
Volkswagon Beetle	2.0	31
Honda Civic HX	1.6	43
Jaguar XJR	4.0	22
Ford Escort	2.0	37
Bentley Continental	6.8	16
Nissan Altima	2.4	31
Porsche Boxster	2.5	26
Toyota Paseo	1.5	41
Mercedes-Benz SL600	6.0	19



Variable

- A quantity whose value may change or vary.
 - What are the two variable quantities in the Fuel Economy problem?
 - Solution: Engine Size and Miles per Gallon



Components of Variable

- Physical Quantity: qualitative value giving context - what is changing?
- Numerical Quantity: quantitative value - what is number or amount?
 - What are the physical quantity and numeric quantity of the variables in the Fuel Economy problem?
 - Solution: Engine size is physical quantity, number of liters is numeric quantity



Relation

- A set of ordered pairs (x,y) in which the value of y is “carried back” or related to the matching x .
 - 5 **association** types for 2 variables
 - Causation: X causes Y
 - Causation: Y causes X
 - Circular Loop: X and Y cause each other
 - Confounding: Z influences X and Y
 - Coincidence: X and Y have no effect on each other



Association versus Causation

- Which association is represented in the Fuel Economy problem?
 - Solution: Causation – increased engine size causes lower fuel efficiency
- If two variables are associated, does one always cause the other?
 - Solution: No, we can randomly assign two variables to get a relationship



Scatter Plot

- Independent Variable: x-axis
- Dependent Variable: y-axis
- Provides a graphic representation of the relationship between two variables

- Use Derive to make a scatter plot of the Fuel Economy data (100 cases)

Miles per Gallon

70

60

50

40

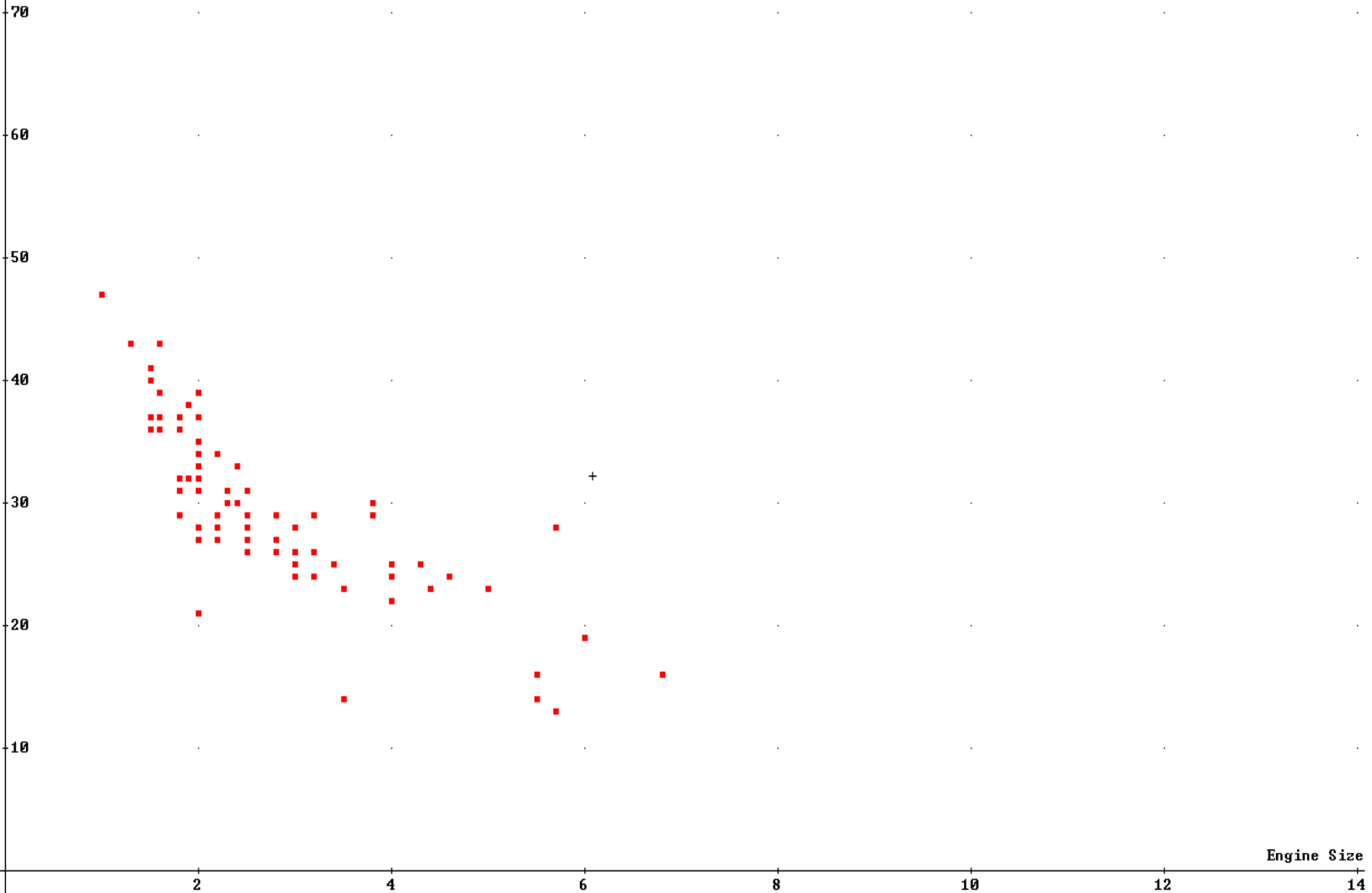
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Engine Size

14





Line of Best Fit-Approximation

- Find a line that represents the trend of the data
 - What point should the best-fit line pass through?
 - Solution: Midpoint of the data
 - How can we use a ruler to determine a line that best fits the data?
 - Solution: Place ruler on the midpoint, vary slope of ruler to best match data trend

Miles per Gallon

70

60

50

40

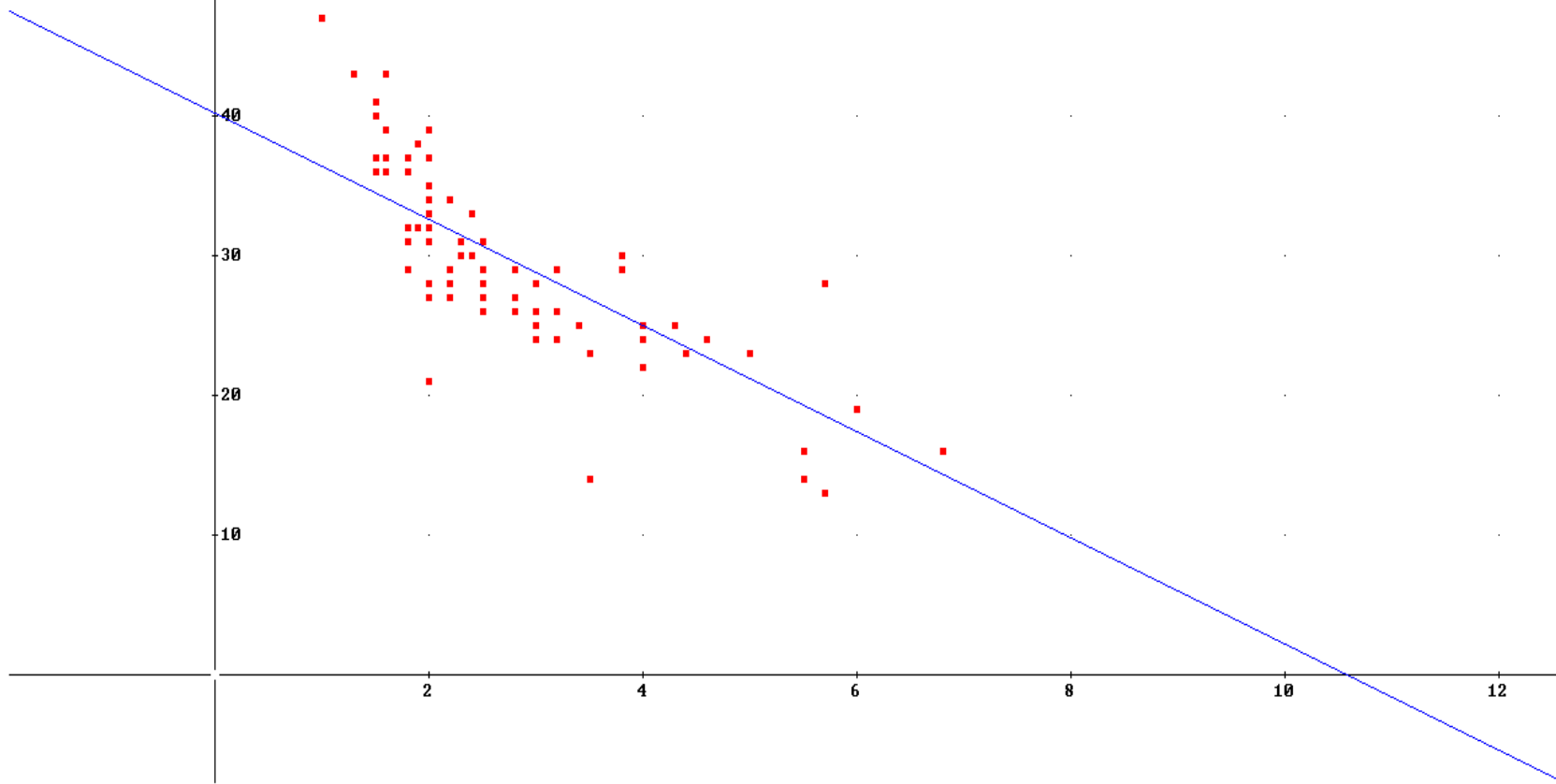
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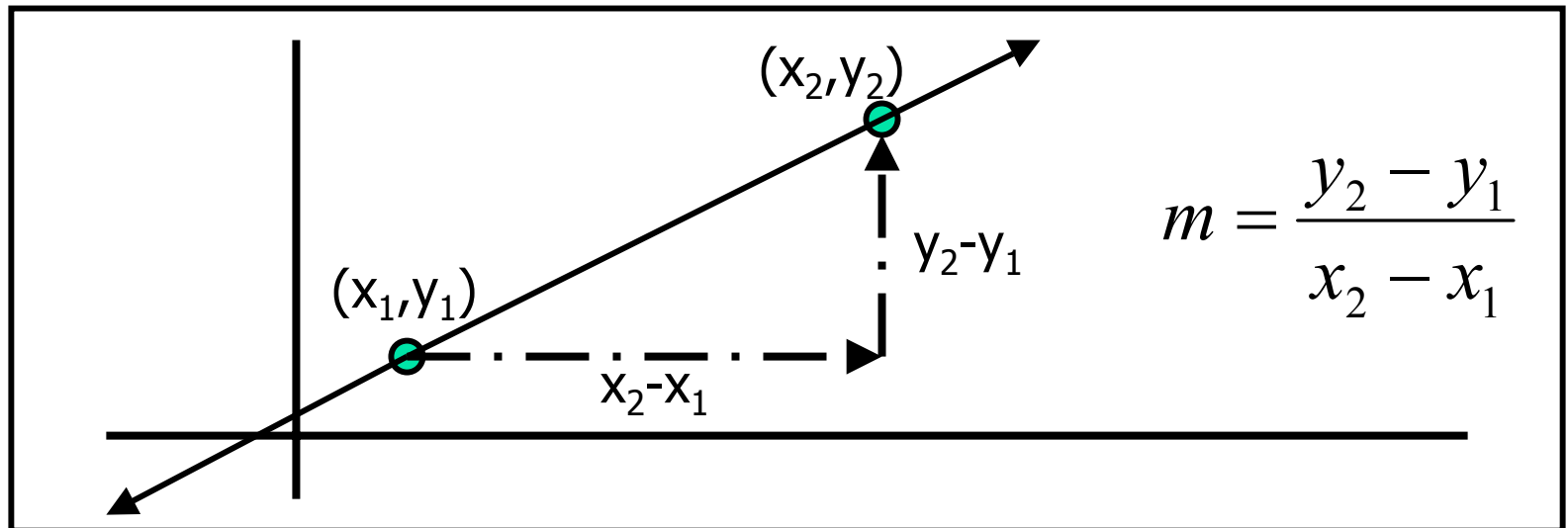
Engine Size

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Linear Relationship-Algebraic

- The line of best fit can be determined from 2 characteristics – a point and the slope. What is slope?





Point-Slope Form of a Line

- Convert slope formula into equation for a line. Let (x, y) be arbitrary point on line and (x_1, y_1) be a given point on line.

$$m = \frac{y - y_1}{x - x_1} \longrightarrow y - y_1 = m(x - x_1)$$

- Apply this to Fuel Economy problem where $(4, 25)$ and $(0, 40.2)$ are two points on the best fit line.



Slope-Intercept Form of Line

- Solve for y in the Point-Slope Form

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = m \cdot x - m \cdot x_1$$

$$y = m \cdot x + (-m \cdot x_1 + y_1)$$

$$y = m \cdot x + b$$