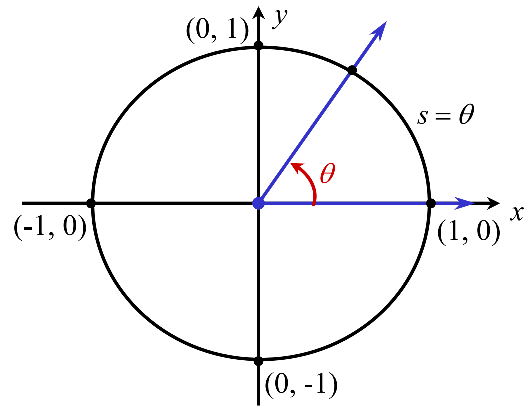
The **unit circle** is a circle whose radius is 1 and whose center is at the origin.

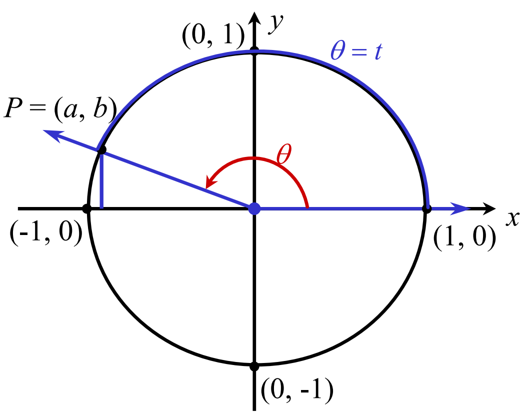
Since $r = 1$:

$$s = r\theta$$

becomes

$$s = \theta$$





$P = (a, b)$ the point on the unit circle that corresponds to t.

The **sine function** matches t with the y-coordinate of P

$$\sin t = b$$

The **cosine function** matches t with the x-coordinate of P

$$\cos t = a$$

Let t be a real number and let $P = \left(\frac{1}{4}, -\frac{\sqrt{15}}{4}\right)$ be the point on the unit circle that corresponds to t. Find the exact value of the six trigonometric functions.

$$(a,b) = \left(1/4, -\sqrt{15}/4\right)$$

$$\sin t = b = -\frac{\sqrt{15}}{4} \qquad \cos t = a = \frac{1}{4}$$

$$(a,b) = \left(1/4, -\sqrt{15}/4\right)$$

$$\tan t = \frac{b}{a} = \frac{-\sqrt{15}/4}{1/4} = -\sqrt{15}$$

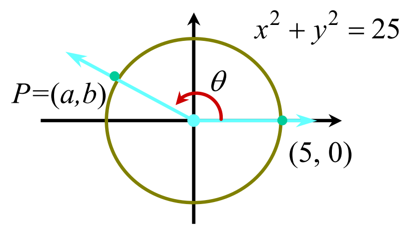
$$\csc t = \frac{1}{b} = \frac{1}{-\sqrt{15}/4} = -\frac{4}{\sqrt{15}} = -\frac{4\sqrt{15}}{15}$$

$$\sec t = \frac{1}{a} = \frac{1}{1/4} = 4$$

$$(a,b)=\left(\tfrac{1}{4},-\tfrac{\sqrt{15}}{4}\right)$$

$$\cot t=\frac{a}{b}=\frac{\frac{1}{4}}{-\frac{\sqrt{15}}{4}}=-\frac{1}{\sqrt{15}}=-\frac{\sqrt{15}}{15}$$

Given that $\sec\theta=\frac{5}{-2}$ and $\sin\theta>0,$ find the exact value of the remaining five trigonometric functions.



$$\sec\theta=\frac{5}{-2}=\frac{r}{a},\text{so } r=5, a=-2$$

$$a^2+b^2=r^2 \text{ with } b>0 \text{ since } \sin\theta=\frac{b}{r}>0$$

$$(-2)^2+b^2=5^2$$

$$4+b^2=25$$

$$b^2=21$$

$$b=\sqrt{21}$$

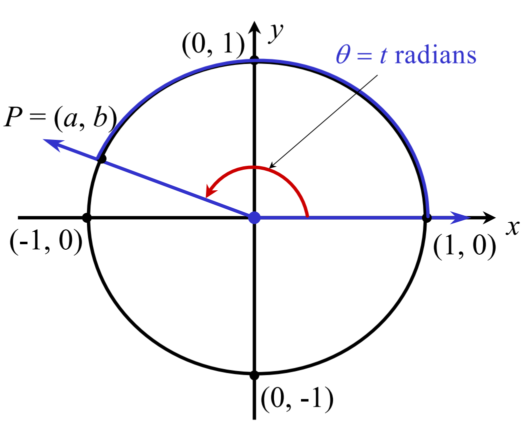
$$a=-2,\ b=\sqrt{21},\ r=5$$

$$\sin\theta=\frac{b}{r}=\frac{\sqrt{21}}{5} \qquad \cos\theta=\frac{a}{r}=\frac{-2}{5}$$

$$\tan\theta=\frac{b}{a}=\frac{\sqrt{21}}{-2}=-\frac{\sqrt{21}}{2}$$

$$\csc\theta=\frac{r}{b}=\frac{5}{\sqrt{21}}=\frac{5\sqrt{21}}{21}$$

$$\cot\theta=\frac{a}{b}=\frac{-2}{\sqrt{21}}=-\frac{2\sqrt{21}}{21}$$



The domain of the sine function is the set of all real numbers.

The domain of the cosine function is the set of all real numbers.

The domain of the tangent function is the set of all real numbers except odd multiples of $\pi/2$ (90°).

The domain of the secant function is the set of all real numbers except odd multiples of $\pi/2$ (90°).

RANGE OF THE TRIGONOMETRIC FUNCTIONS

Let $P = (a, b)$ be the point on the unit circle that corresponds to the angle θ. Then, $-1 \leq a \leq 1$ and $-1 \leq b \leq 1$.

$$\sin\theta = b \qquad \cos\theta = a$$

$$-1 \leq \sin\theta \leq 1 \qquad -1 \leq \cos\theta \leq 1$$

$$|\sin\theta| \leq 1 \qquad |\cos\theta| \leq 1$$

$$|\csc\theta| = \frac{1}{|\sin\theta|} = \frac{1}{|b|} \geq 1$$

$$\csc\theta \leq -1 \quad \text{or} \quad \csc\theta \geq 1$$

$$|\sec\theta| = \frac{1}{|\cos\theta|} = \frac{1}{|a|} \geq 1$$

$$\sec\theta \leq -1 \quad \text{or} \quad \sec\theta \geq 1$$

$$-\infty < \tan\theta < \infty$$

$$-\infty < \cot\theta < \infty$$

A function f is called **periodic** if there is a positive number p such that whenever θ is in the domain of f, so is $\theta + p$, and

$$f(\theta + p) = f(\theta)$$

Periodic Properties

$$\sin(\theta + 2\pi) = \sin\theta \qquad \csc(\theta + 2\pi) = \csc\theta$$

$$\cos(\theta + 2\pi) = \cos\theta \qquad \sec(\theta + 2\pi) = \sec\theta$$

$$\tan(\theta + \pi) = \tan\theta \qquad \cot(\theta + \pi) = \cot\theta$$

Theorem Even-Odd Properties

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$