Quadratic Functions

Names: ***KEY***

(8 communication points)

About this Laboratory

A quadratic function in the variable x is a polynomial where the highest power of x is 2. We will explore the domains, ranges, and graphs of quadratic functions. After completing this laboratory, you should be able to graphically approximate and algebraically determine such features as the intercepts, vertex, and shape of a quadratic function. You should also be able to conceptually understand how these features relate to an application problem.

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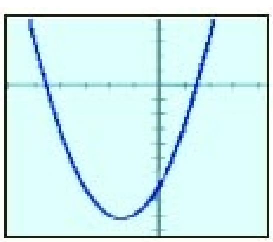
Enter the numerical coefficients of $f(x) = x^2+3x-7$ into the boxes for **f(x) =** .

Select **Graph**. You will see the graph of the function. If possible, you will see the function in factored form and the coordinates of the vertex. The factored form will allow you to easily determine the intercepts if they exist.

Show your *algebraic* work here to compute the x-intercepts, y-intercept and vertex. If necessary, refer to your notes or book and clearly label each result. Algebraically determine the intercepts (as decimals to hundredths) and the vertex coordinates (as improper fractions) of your function.

Sketch your function here (1 point):



Compute the x-intercepts and y-intercept. Clearly label each (6 points).

x-int : $0 = x^2+3x-7$

$$x = \frac{-3 \pm \sqrt{9+28}}{2} = 1.54, \; -4.54$$

(1.54, 0) and (-4.54, 0)

y-int: $y = (0)^2+3(0)-7 = -7$

(0, -7)

Compute the vertex (4 points).

$$\frac{-b}{2a} = \frac{-3}{2} \qquad f\left(\frac{-3}{2}\right) = \left(\frac{-3}{2}\right)^2 + 3\left(\frac{-3}{2}\right) - 7 = \frac{-37}{4} \qquad \left(\frac{-3}{2}, \frac{-37}{4}\right)$$

A quadratic function is defined by a second-degree polynomial in one variable of the form $f(x) = ax^2+bx+c$. The parabola will open up when there is a minimum functional value and it will open down when there is a maximum functional value. If the leading coefficient is positive, there will be a minimum functional value. If the leading coefficient is negative, there will be a maximum functional value.

Which do you have a maximum or minimum value associated with the function (1 point)?
Minimum

What is this value (1 point)? ***-37/4*** (the y-coordinate)

Where does this value occur (1 point)? ***-3/2*** (the x-coordinate)

What is the domain of your function (2 points)? (Remember that this is the set of x-values that may be plugged into your equation.) ***The set of all real numbers***

What is the range of your function (2 points)? (Remember that this is the set of y-values that may be obtained from the x-values that are plugged into your equation.) ***[-37/4, ∞)***

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Enter the information to let $f(x) = -x^2-5x-2$.

Select **Graph**.

Show your *algebraic* work here to compute the x-intercepts, y-intercept and vertex. If necessary, refer to your notes or book and clearly label each result. Algebraically determine the intercepts (as decimals to hundredths) and the vertex coordinates (as improper fractions) of your function.

Compute the intercepts and vertex.
Clearly label each (9 points).

x-int: $0 = -x^2-5x-2 = x^2+5x+2$

$$x = \frac{-5 \pm \sqrt{25-8}}{2} = -4.56,\ -0.44$$

(-4.56, 0) and (-0.44, 0)

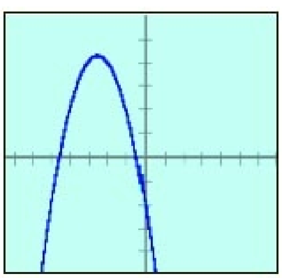
y-int: $y = -(0)2-5(0)-2 = -2$
(0, -2)

Vertex: $\frac{-b}{2a} = \frac{5}{-2} = -2.5$

$$\left(\frac{-5}{2}, \frac{17}{4}\right)$$

$$f\left(\frac{-5}{2}\right) = -\left(\frac{-5}{2}\right)^2 - 5\left(\frac{-5}{2}\right) - 2 = \frac{17}{4}$$

Sketch your function here (1 point):



Which do you have a maximum or minimum value associated with the function (1 point)? ***Maximum***

What is this value (1 point)? ***17/4***

Where does this value occur (1 point)? ***-5/2***

What is the domain of your function (2 points)?
The set of all real numbers

What is the range of your function (2 points)? ***(-∞, 17/4]***

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Let $f(x) = 2x^2-6x+10$.

Select **Graph**.

Show work to determine the y-intercept and vertex. Show algebraic work to explain why there are no x-intercepts. Give the domain, range, and axis of symmetry. If necessary, refer to your notes or book and clearly label each result. Algebraically determine the intercepts (as decimals to hundredths) and the vertex coordinates (as improper fractions) of your function.

Clearly label each (15 points).

x-int: $0 = 2x^2 - 6x + 10$

$$x = \frac{6 \pm \sqrt{36-80}}{4}$$

Since the discriminant is negative, the graph has no x-intercepts.

vertex: $\frac{-b}{2a} = \frac{6}{4} = \frac{3}{2}$

$$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 10 = \frac{11}{2}$$

$\left(\frac{3}{2}, \frac{11}{2}\right)$

y-int: $y = 2(0) - 6(0) + 10 = 10$

$(0, 10)$

Axis of Symmetry: $x = \frac{3}{2}$

Domain: $(-\infty, \infty)$

Range: $\left[\frac{11}{2}, \infty\right)$

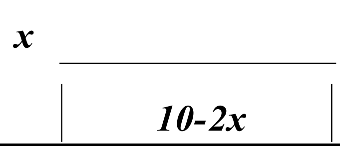
CP2

Use what you have learned about quadratic equations to solve the following problem:

A rain gutter is to be made of a metal sheet that is 10 inches wide by turning up the edges 90°. What depth will provide the maximum cross sectional area to allow the most water flow? What is the maximum cross sectional area?

Let x be the depth of the gutter and sketch a picture of the gutter with the length and depth labeled in terms of x (2 points).

x = depth of the gutter



Develop a formula for cross sectional area by letting x define the unknown quantity of depth that is to be the independent value.

f(x) = ***$-2x^2+10x = x(10-2x)$*** (2 points)

The order of the following steps is important:
Select **Draw Gutter** in the applet. Fill in the rectangle to set the sheet width in the applet as 10 and **press Enter** on the keyboard, then choose inches as the unit being used. Select **Refresh**.

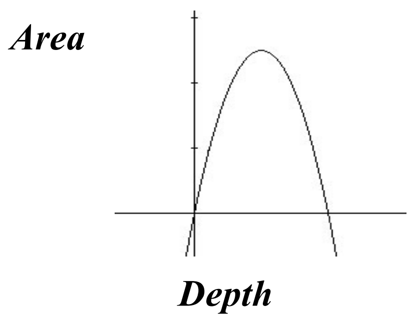
Point the arrow to the model and drag until you have depths that are approximately equal to the values listed in the table below. Select **Capture Data Point** to choose and then record the area values by filling in the table below (1 point).

depth (in.)	Area (in.^{2})
0	***0***
1	***8***
2	***12***
3	***12***
4	***8***
5	***0***

What happens to the cross sectional area as the depth size increases; give a complete answer (2 points).

It increases and then decreases. Note that there is symmetry in the table values.

Select **Plot Data Points** and then enter the area expression from the previous page into the equation box. Select **Plot Function**. Sketch a "complete graph" showing such things as intercepts and turning points (1 point). (Label the axes with appropriate words such as area, volume, ...)



What does the x variable represent in this example (2 points)? ***Depth of gutter***

What does the f(x) value represent in this example (2 points)? ***Cross sectional area***

What is the domain of the **function** (2 points)? ***(-∞, ∞)***
(Without respect to the problem.)

What is the range of the **function** (2 points)? ***(-∞, 12.5]***
(Without respect to the problem.)

What x values **make sense** for the **problem** we are solving (2 points)? ***(0, 5)***
These values make up the *restricted domain*. Hint: Some values for x and y on the graph do not make sense for the gutter problem. Make a guess and come back to check this answer later.

What y values **make sense** for the **problem** we are solving (2 points)? ***(0, 12.5]***
These values make up the *restricted range*. Hint: Some values for x and y do not make sense for the gutter problem. Make a guess and come back to check this answer later.

Solve to find the exact value of each x-intercept (1 point per intercept).

x-intercept: (***0***, 0)

x-intercept: (***5***, 0)

Describe the real life meaning or insight to the problem that the x-intercepts provide; that is, how do the coordinates relate to the numbers found in your gutter model and the restricted domain (8 points)?

Given a gutter 10 inches wide, 0 inches and 5 inches represent the "bounds" of the least and most measurement of the depth. The x-intercepts help us determine where the cross sectional area would be zero. Since the values for f(x) are negative for x values less than 0 and greater than 5, they also help us find the restricted domain.

Solve to find the exact coordinates of the vertex (2 points). (***2.5*** , ***12.5***)

Describe the real life meaning of the vertex coordinates (4 points).

When the depth of the gutter is 2.5", the maximum cross sectional area of 12.5 square inches is obtained.

What depth will provide the maximum cross sectional area to allow the most water flow (2 points)? (Include dimensional units such as inches, centimeters, square inches, ...)

2.5 inches

What cross sectional dimensions will provide the maximum area to allow the most water flow (2 points)? (Include dimensional units such as inches, centimeters, square inches, ...)

2.5" x 5"

What is the maximum cross sectional area (2 points)? (Include dimensional units such as inches, centimeters, square inches, ...)

12.5 square inches