Trigonometric Graphs

Names:____

About this Laboratory

In this laboratory, we will examine trigonometric functions and their graphs. Upon completion of the lab, you should be able to quickly sketch such functions and determine such characteristics as period and amplitude. You should also be able to determine whether the function has been shifted, reflected, stretched or shrunk as compared to the graph of one of the six trigonometric functions discussed in a previous laboratory.

****

When using the unit circle: sin(t) = ____________ cos(t) = ____________

Periodic Definition. Fill in the following using your class notes or text:

A function is called periodic if there is a positive number p such that whenever θ is in the domain of f, so is $\theta + p$ and $f(\theta + p) = f(\theta)$. If there is a smallest such number p, then p is called _________________________

Generally speaking, a function is called **periodic** if it repeats itself within a certain interval on the *x*-axis.

Look at the graph of f(t) = sin(t) on the **right**. The sine function repeats itself every 2π units.(2π radians or 360 degrees if you are looking at the circle)

For a real number t:
If *f(t) = sin(t)*, $0 \le t \le$___________________ gives one complete cycle of the sine graph.
(As an irrational number)

Select **the radio button** to graph the cosine function and name one cycle.
$0 \le t \le$_______. (As an irrational number)

Select **the radio button** to graph the tangent function and name one cycle.
$-\pi / 2 < t <$ _________. (As an irrational number)

The fundamental *period* of the:
Sine function is:___________ Cosine function is:_________Tangent function is:________
(As irrational numbers)

CP2 ***

We will now examine changes to the graphs of the Sine Cosine, and Tangent functions.

Trigonometric Graphs

For the remainder of this laboratory, once your graph appears, you may click on the screen and use the computer arrow keys to trace. You may also use the applet to scroll and zoom in and out as needed

Select **Graph** for *f(x) = a*sin(x)*. Notice that a slider bar appears for a, the coefficient. Make sure that a is initially set to 1. Name the maximum and minimum values for *f(x)*.(The dependent value)

____________ $\leq$ *sin(x)* $\leq$ __________.

On the same axis, graph the following: (only on the computer screen) Adjust the value for a as needed.

g(x) = 3 sin(x) ____________ $\leq$ *sin(x)* $\leq$ __________.

h(x) = (3/2) sin(x) ____________ $\leq$ *sin(x)* $\leq$ __________.

k(x) = (-2) sin(x) ____________ $\leq$ *sin(x)* $\leq$ __________.

Write a statement to generalize your results.

Given *f(x) = a*sin(x)*, |*a*| is called the **amplitude** of the graph. If |*a*| < *1*, we will say that we have a **shrink** (when compared to *f(x) = sin(x))*. If |*a*| > *1*, we will have a **stretch**. Also, if a is negative, then there is a **reflection**.

Adjust the value for a as needed and label each of the following with the adjectives: **shrink, stretch**, or **reflection**. Graph to check your results. If you missed something, correct your work. (You may need to use more than one adjective and compare your functions to f(x) = sin (x).)

g(x) = 5 sin(x) ________________________

h(x) = (- ½) sin(x) ________________________

k(x) = (4/3) sin(x) ________________________

Change the function on this page to f(x) = a* cos(x) . Next select **Graph**.

Trigonometric Graphs

Notice that a slider bar appears for a, the coefficient. Make sure that a is initially set to 1. Name the maximum and minimum values for *f(x)*.(The dependent value)

____________ ≤ *cos(x)* ≤ __________

Apply the same adjectives to the following functions and graph to check your results. Compare these to the graph of f(x) = cos (x).

g(x) = 2 cos(x) ________________________

h(x) = (-3/2) cos(x) ________________________

k(x) = (.2) cos(x) ________________________

Change the function on this page to f(x) = a* tan(x) . Next select **Graph**.

Notice that a slider bar appears for a, the coefficient. Make sure that a is initially set to 1. Name the maximum and minimum values for *f(x)*.(The dependent value)

____________ < *tan(x)* < __________

Apply the same adjectives to these functions and graph to check your results:

g(t) = (½) tan(t) ________________________

h(t) = 2 tan(t) ________________________

k(t) = (-10) tan(t) ________________________

Stretch and **shrink** have slightly different meanings when applied to the sine and cosine function than when used for the tangent function.
Explain:

CP3

Trigonometric Graphs

Next, we will examine vertical shifts. We will denote a vertical shift as **U(#) meaning upward # units , and D(#) as downward # units**.

Change the function on this page to f(x) = a* sin(x) +b . Next select **Graph**.
Notice that a slider bar appears for a and b. Make sure that a is initially set to 1.

Graph each *f(x) = sin(x)*, *f(x)* = ***sin(x) + 2***, and f*(x)* = ***sin(x) -2*** on the same axis in the box below. Be sure to identify each function.

What in the equation indicates that a vertical shift will occur?

Compare the following functions to the graphs of Sine, Cosine, or Tangent by using the following adjectives: **shrink**, **stretch**, **reflection**, **U(#)**, and/or **D(#)**. Use the computer to graph each function to check your results. (Fill in the appropriate # and use as many adjectives as needed.)

f(x) = 3 cos(x) + 4 ____________________________

g(x) = (½) sin(x) - 3 ____________________________

h(x) = (3/2) tan(x) + 1 ____________________________

k(x) = (-2) cos(t) + (.1) ____________________________

Trigonometric Graphs

CP4

We can algebraically determine the beginning point and ending point for the cycle of a function such as *f(x) =3 sin (x - π/4)*.

Remember that for f(x) = sin(x) we found that $0 \le x \le 2\pi$ gave one cycle.
Similarly for *f(x) = 3sin (x - π/4)*, we can let $0 \le (x - \pi/4) \le 2\pi$ because we know that the period for the Sine function is 2π. Notice that the coefficient "3" is not needed for this analysis.

Example of Solving an Inequality for x:

If $0 \le (x - \pi/4) \le 2\pi$, then $\pi/4 \le x \le 2\pi + \pi/4$ or $\boldsymbol{\pi/4 \le x \le 9(\pi/4)}$.

So these values for x show one complete cycle of *f(x) = 3sin(x - π/4)* . Therefore, the exact starting and stopping point of one cycle for *f (x) = 3sin(x - π/4)* is expressed in the following inequality: $\pi/4 \le x \le 9(\pi/4)$.

Notice from previous work with polynomials that we would expect a **horizontal** shift *π/4* units to the _________________________ Left or Right (Choose one) when compared to the Sine function.

You may wish to graph to verify. (Use Pi for π.) Type $f(x) = a * \sin(x - \frac{Pi}{b})$, select Graph and adjust the values as needed.

Now you try: Show work in the box provided to algebraically determine an *exact* starting and stopping point with respect to the *phase* (horizontal) shift of *f(x) = cos(x + π/3)*

We expect f(x) = cos(x + π/3) to have a __________ shift, _______ units to the ________ when compared to the Cosine function. You may wish to graph to verify.

Trigonometric Graphs

Now consider *f(x) = sin(2x)*. $0 \le 2x \le 2\pi$ gives one complete picture of the graph, so

____________$\le$ x $\le$________. You might call this a **squeeze**. Why? (Try graphing *f(x) = sin(a*x)* and explore for different values of a.)

Now try the following:

A) For each, use **stretch**, **shrink**, **squeeze**, **reflect, U(#)**, **D(#)**, **H(right)**, **H(left)**. Give values for #. Use as many of the adjectives as needed.

B) Name the endpoints of the interval on the x-axis on which one complete cycle of the graph will appear. For the Tangent function you will consider the inequality boundaries with respect to the Vertical Asymptotes at x = - π/2 and x = π/2. (**Show your work**.)

g(x) = (-3/4) cos(x - π/4)

h(x) = 3 tan (2x)

k(x) = 2 sin(2x - π/3) -1

CP5

Give an equation to match the graph: ___________________________________

CP6

Give an equation to match the graph: ___________________________________