Trigonometric Functions

Names:____

About this Laboratory

In this laboratory, we will examine six trigonometric functions and their graphs using a unit circle approach. Upon completion of the lab, you should be able to quickly sketch each function and determine its domain and range.

CP 1
The applet on this page represents the unit circle.

The unit circle has a radius = ___________________ and center at _____________ .

Let t be a real number that represents the arclength from the point (1,0) to the point P. In other words, t would be the length of a string wrapped around the circumference with one end on the point (1,0) and the other end on P. In this example, we will not consider the possibility that you will wrap the circle more than one time.

Begin by dragging the point P = (x,y) to coincide with A = (1,0). Now drag the point P along the circumference of the circle and choose a place to stop before you complete one wrap. Draw what you see in the box provided below and shade in the arclength on your sketch beginning at A and ending at P.

The sine function associates with t the y-coordinate of P, therefore sin (t) = y
The cosine function associates with t the x-coordinate of P, therefore cos (t) = x
So P = (x,y) = (Cos (t) , Sin (t))

What color on the graph of the unit circle represents x? ___________

What color on the graph of the unit circle represents y? __________

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Continue to drag P along the circle in both directions and watch the values for x and y.
What do you notice about the values for x and y? ________________________________
(Think about the x and y-values that can be obtained on the unit circle.)

Now sketch the unit circle **four times**. One per sketch, mark as an arc length:
$t = \pi/2$ $t = \pi$ $t = 3\pi/2$ $t = 2\pi$

Remember that π is an irrational number ≈ 3.14. (Note that the applet uses both positive and negative multipliers so you will need to think a little to get what you need.)
Under each sketch, give the **coordinates** of P; this designates the "**end** of the arc".

$t = \pi/2$

$t = \pi$

$t = 3\pi/2$

$t = 2\pi$

Use your sketches to answer the following, then use the computer to check your answer.

$\sin(\pi/2)$ = _______________
$\cos(\pi/2)$ = _______________

$\sin(\pi)$ = _________________
$\cos(\pi)$ = _________________

$\sin(3\pi/2)$ = ______________
$\cos(3\pi/2)$ = ______________

$\sin(2\pi)$ = ________________
$\cos(2\pi)$ = ________________

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CP 2

Select the **radio button** next to Sine.

On the **left** is the **unit circle**. Notice that you can drag along the circumference and trace out an arclength. When the length of the arc from (1,0) is zero then t = 0. Therefore:

cos (0) = ___________ sin (0) = ___________

On the **right** is the **graph of the Sine function**. Notice that as you drag along the circumference of the unit circle the arclength is also traced along the x-axis of the Sine function. Therefore on the graph of the Sine function:

t, the independent variable, is associated with the ______________ axis.

sin (t), the dependent value, is associated with the ______________ axis.

So, when the arclength is zero, this means that t = 0, and the sin (t) = 0.

DOMAIN of Sine on the Unit Circle:
Remember that t is the distance moved from (1, 0) on the circle. If we think of t as the distance walked as we move around a circle, beginning at (1, 0) you can see that while walking in a counter-clockwise direction, t can be any real number greater than zero. If we walk in a clock-wise direction, t can be any real number less than zero. Since we started at t=0, we may determine that t can be any real number.

RANGE of Sine on the Unit Circle:
For a point P = (x,y) on the *unit circle*, **y** = sin (t), therefore _____ ≤ sin (t) ≤ _________.
(Think about the y-values that can be obtained on the unit circle.)

DOMAIN of Sine function on the Graph:
On the **right** (on the *graph of the Sine function*) using what we learned in the unit circle, we find that t can be any real number (we are viewing the graph for $0 \le t \le 2\pi$.)

RANGE of Sine function on the Graph:
If **y** = sin (t), ______________ ≤ sin (t) ≤ _________ .
(Think about the y-values that can be y-coordinates for points on the graph of the Sine Function.)

By using either the unit circle or the graph of the sine function, give the **domain and range** for y = sin (t). ___________________________________

Now unwrap the arclength so that there is no pink showing on the circumference.

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Drag around the *unit circle* one time. t = _______________ (as an irrational number)

Sketch in the box provided the part of the Sine function that correlates with one full revolution around the unit circle. Your sketch represents one cycle or one period for y = sin (t).

The end value for t in the sketch is t =_____________ (In terms of π)

So the period of the Sine function is _____________. This is the t-value for 1 revolution around the unit circle and the t-value for one complete "cycle" on the graph.

Select **the radio button next to Cosine.**

On the **left** is the **unit circle**. Notice that you can drag along the circumference and trace out an arclength. When the length of the arc from (1,0) is zero then t = 0. Therefore:

cos (0) = ___________ sin (0) = ___________

On the **right** is the **graph of the Cosine function**. Notice that as you drag along the circumference of the unit circle the arclength is also traced along the x-axis of the Cosine function. Therefore <u>on the graph of the Cosine function</u>:

t, the independent variable, is associated with the ______________ axis.
cos (t), the dependent value, is associated with the ______________ axis.

So, when the arclength is zero, this means that t = 0, and the cos(t) = 1.

DOMAIN of Cosine on the Unit Circle:
Remember that t is the distance moved from (1, 0) on the circle. If we think of t as the distance walked as we move around a circle, beginning at (1, 0) you can see that while walking in a

counter-clockwise direction t can be any real number greater than zero. If we walk in a clock-wise direction, t can be any real number less than zero. Since we started at t=0, we may determine that t can be any real number.

RANGE of Cosine on the Unit Circle:
For a point P=(x,y) on the *unit circle*, **x** = cos (t), therefore ______ ≤ cos (t) ≤ _________ .
(Think about the x-values that can be obtained on the unit circle.)

DOMAIN of the Cosine function on the Graph:
On the **right**, on the *graph of the Cosine function*, from previous work, we know that t can be any real number (we are viewing the graph for $0 \le t \le 2\pi$.)

RANGE of the Cosine function on the Graph:
If y = cos (t), _____________ ≤ cos (t) ≤ ________ .
(Think about the y-values that can be y-coordinates for points on the graph.)

By using either the unit circle or the graph of the cosine function, give the **domain and range** for y = cos (t). _____________________________________

Now unwrap the arclength so that there is no pink showing on the circumference.

Drag around the *unit circle* one time. t = _______________ (as an irrational number)

Sketch in the box provided the part of the Cosine function that correlates with one full revolution around the unit circle. Your sketch represents one cycle or one period for y = cos (t).

The end value for t in the sketch is t =____________ (In terms of π)

So the period of the Cosine function is _________________. This is the t-value for 1 revolution around the unit circle and the t-value for one complete “cycle” on the graph.

Select **the radio button next to Tangent.**

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As you drag along the unit circle on the left watch on the graph of the tangent function to determine when the graph begins to "repeat itself".

When beginning at the point (1,0) on the unit circle, how much of the circle is wrapped when the graph of the tangent function on the right begins to repeat? ___________________

What is the value of t at this point? _____________________ (In terms of π)
The pattern for the tangent function repeats every ________ units; therefore the **period of the Tangent** function is π.

Recall that $\tan(t) = \frac{\sin(t)}{\cos(t)}$. Therefore $\frac{\sin t}{\cos t} = \frac{\tan t}{1}$.

Also remember that P = (x,y) =(Cos (t), Sin(t))

Drag P to a point in quadrant I. In the box below, sketch your circle and fill in right triangles. Use properties of similar triangles to show that the blue segment represents Tan(t) based on the proportion given above.

Because division by zero is undefined, the graph of the Tangent function will be undefined for some values on the real number line.

Since $\tan(t) = \frac{\sin(t)}{\cos(t)}$, the points where the Tangent function are undefined are where cos (t) = ___________.

What should happen to the tangent function when cos(t)=0?_________________________

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In the domain from (0, 2π], for which values of t is cos (t) = 0?______________________
(You may flip back to previous pages to help to answer the question.)

Evaluate the following.

tan (π/2) = $\dfrac{\sin(\frac{\Pi}{2})}{\cos(\frac{\Pi}{2})} = \dfrac{1}{0}$, which is undefined

tan (π) =

tan (3π/2) =

tan (2π) =

Notice that the graph of the Tangent function has Vertical Asymptotes at t = π/2 and t = 3π/2.

In the box provided, show mathematically how you can find the next 4 vertical asymptotes by using the knowledge that x = π/2 and x = 3π/2 are vertical asymptotes and that the period of the function is π.(Use π in your answers.)

For example 3π/2 + π = 5π/2. So x = 5π/2 is a vertical asymptote for the tangent function.

What are the domain and range for the tangent function? You may use words in your answer.

CP 3 ***

Graph sin (x) and $\frac{1}{\sin(x)}$ on the same set of axes. Note that $\frac{1}{\sin(x)}$ = csc (x).

How do the Vertical asymptotes for the Cosecant function relate to the domain values of the Sine function? (When will a Vertical Asymptote occur?)

Sketch the Sine and Cosecant functions on the same set of axes. Include the asymptotes on your sketch.

CP 4*

Graph cos (x) and $\frac{1}{\cos(x)}$ on the same set of axes. Note that $\frac{1}{\cos(x)}$ = sec (x).

How do the Vertical asymptotes for the Secant function relate to the domain values of the Cosine function? (When will a Vertical Asymptote occur?)

Sketch the Cosine and Secant functions on the same set of axes. Include the asymptotes on your sketch.

CP 5

Graph tan (x) and $\frac{1}{\tan(x)}$ on the same set of axes. Note that $\frac{1}{\tan(x)}$ = cot (x).

How do the Vertical asymptotes for the Cotangent function relate to the domain values of the Tangent function? (When will a Vertical Asymptote occur?)

Sketch only the **Cotangent** function and **answer the following**:
What are **some** features that you can use to distinguish the Cotangent graph from Tangent graph?