

Rational Functions
© Dr. Laura J. Pyzdrowski

Names: _____

About this Laboratory

A rational function is the ratio of two polynomials. In this laboratory, we will explore the graphs of rational functions. You will be expected to accurately sketch such functions and label all important features on the graph. Before you begin exploring the features of a rational function, you should make sure that there are no common factors in the numerator and denominator. This process of simplification to "reduced form" is necessary for the analysis described in this laboratory. In this laboratory, all answers in decimal form should be within .1 and all asymptotes should be expressed as equations. Exact solutions for intercepts and asymptotes should be found algebraically and confirmed graphically.

CP 1*****

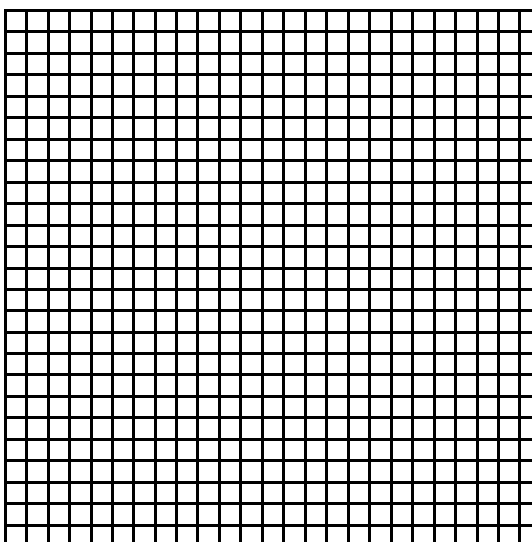
Refer to your class notes or your text and answer the following:

Vertical asymptotes of a rational function are found by _____
_____.

$$\text{Let } f(x) = \frac{(x+1)(x-1)}{(x-1)} = \frac{p(x)}{q(x)}.$$

Sketch a graph of $f(x)$ and fill the values into the table.

x	f(x)
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



Explain why there are no vertical asymptotes on your graph.

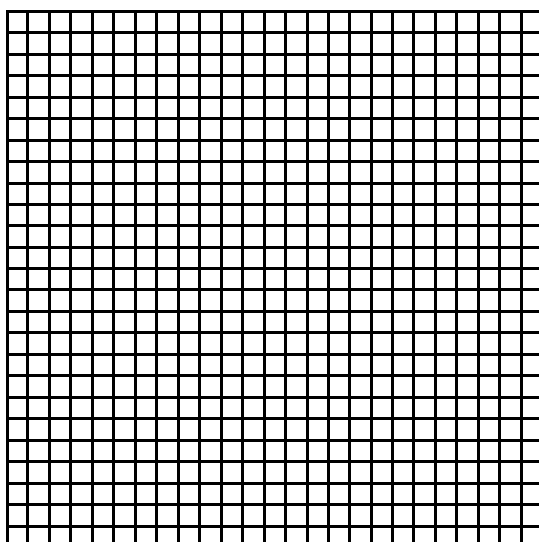
Give the domain of $f(x)$ using interval notation. _____

$$\text{Let } f(x) = \frac{3x + 5}{(x - 1)(x - 4)} = \frac{p(x)}{q(x)}.$$

For the rational function $f(x) = \frac{p(x)}{q(x)}$ in simplest form, what graphical feature occurs for values of x that allow $q(x)$, the polynomial in the denominator, to be equal to zero?

In this example, $q(x) =$ _____.

Graph $f(x)$ and $q(x)$ in the computer viewing window.
Sketch $q(x)$ here. (Label all intercepts.)



What are the x -intercepts for $q(x)$?
(Remember these are **points with two coordinates.**)

In order to approximate these graphically, move the arrow inside the graph window, it will change to a cross hair. Point to and click anywhere inside the viewing window. You can now use the right and left keyboard arrow keys to trace along the graph. The up and down arrow keys will allow you to jump between graphs. This along with the zooming in process will help you to approximate x -intercepts.

Rational Functions
© Dr. Laura J. Pyzdrowski

Algebraically confirm your x-intercepts for q(x) here.

What are the vertical asymptotes for f(x)? (Remember to give **equations of lines.**)

Refer to your class notes or your text and answer the following:

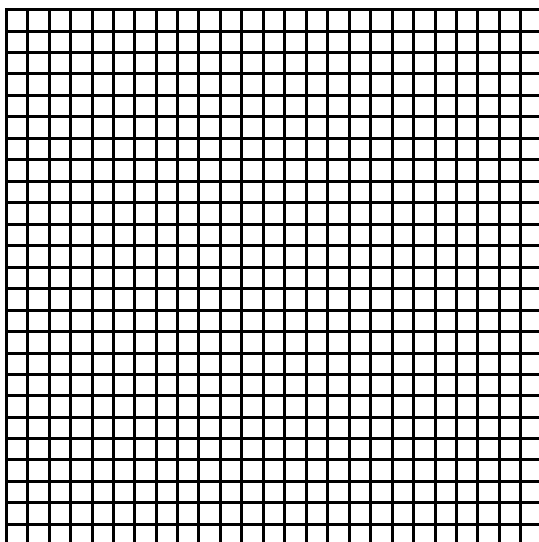
x-intercepts of a rational function are found by _____.

y-intercepts of a rational function are found by _____.

$$\text{Let } f(x) = \frac{3x + 5}{(x - 1)(x - 4)} = \frac{p(x)}{q(x)}.$$

p(x) = _____.

Graph f(x) and p(x) in the viewing window.
Sketch p(x) here. (Label all intercepts.)



What is the x-intercept for p(x)?
(Remember this is a **point.**)

Show your algebraic work here.

What is the x-intercept for f(x)?

Refer to your class notes or your text and answer the following:

What type of end behavior asymptote (horizontal or oblique) do you expect to find for

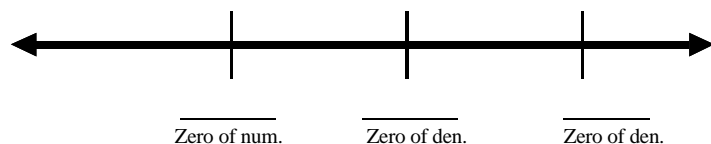
$$f(x) = \frac{3x + 5}{(x - 1)(x - 4)} = \frac{p(x)}{q(x)} \text{ and why? (Be as specific as possible.)}$$

Your end behavior asymptote is $y =$ _____.

Putting it all together:

Determining where the graph of the rational function is above or below the x-axis.

On the number line below (which is not drawn to scale) fill in 3 values: the x-coordinates of the x-intercept(s) and the vertical asymptotes of $f(x) = \frac{3x + 5}{(x - 1)(x - 4)} = \frac{p(x)}{q(x)}$. (You have already found these on previous pages.)



These are the x-coordinates of the x-intercept(s) and the vertical asymptotes for the rational function. The x-intercept(s) and the vertical asymptotes give the places on the graph where there can be a sign change for the y-values of the points on the graph. In other words they help determine where the graph is above or below the x-axis.

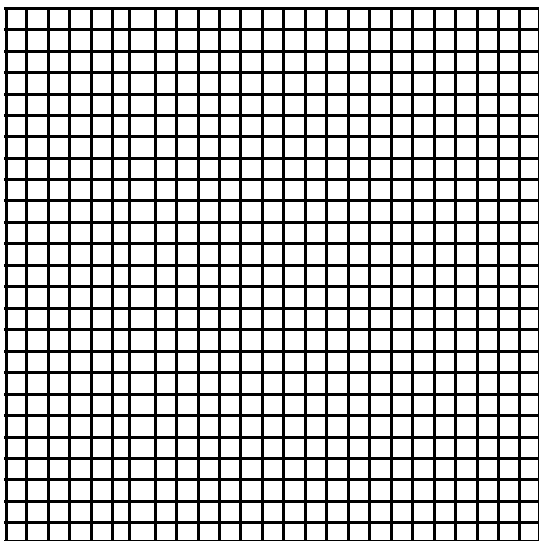
Graph $p(x)$ and $q(x)$ in the viewing window.

Rational Functions
© Dr. Laura J. Pyzdrowski

These points should be on the graph of $f(x) = \frac{3x + 5}{(x - 1)(x - 4)} = \frac{p(x)}{q(x)}$. Zoom and trace if

necessary. Pay attention to the sign of the functional or y-values in the associated region of the domain of the function.

Sketch $f(x) = \frac{3x + 5}{(x - 1)(x - 4)} = \frac{p(x)}{q(x)}$. You should only sketch $f(x)$ showing all intercepts, and asymptotes being careful to be above or below the x-axis as necessary.



Same computer page, new problem *****

Clear all of the functions from the grapher and let $f(x) = \frac{4x + 3}{x - 1}$.

What is the vertical asymptote of $f(x)$? _____ (Equation of a line)

Show your algebraic work here.

Rational Functions
© Dr. Laura J. Pyzdrowski

What is/are the x-intercepts of $f(x)$? _____(point(s))
Show your algebraic work here.

What is the y-intercept of $f(x)$? _____(point)
Show your algebraic work here.

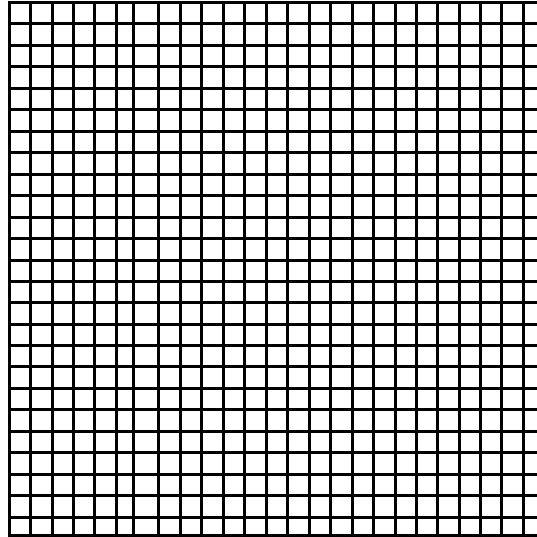
What is the horizontal asymptote of $f(x) = \frac{4x + 3}{x - 1}$? (This is an equation of a line.)

Use long division here to confirm your answer.

$$\mathbf{x - 1 \overline{)4x + 3}}$$

Graph your horizontal asymptote in the computer viewing window and then carefully sketch

$f(x) = \frac{4x + 3}{x - 1}$ here.



Same page, new problem *****

Clear all previous functions and let $f(x) = \frac{2x^2 - 1}{x + 1}$.

What is the vertical asymptote of $f(x)$? _____ (Equation of a line)

Show your algebraic work here.

What is/are the x-intercepts of $f(x)$? _____ (point(s))

Show your algebraic work here.

Rational Functions
© Dr. Laura J. Pyzdrowski

What is the y-intercept of $f(x)$? _____ (point)

Show your algebraic work here.

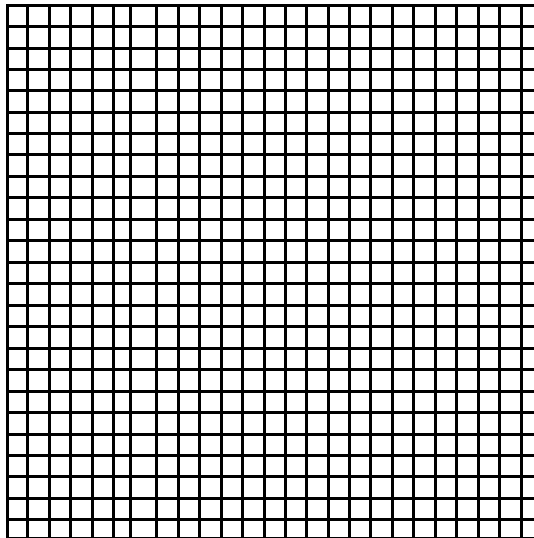
What is the oblique asymptote of $f(x)$? (Equation of a line.) _____

Use long division here to confirm your answer.

$$\mathbf{x + 1 \overline{) 2x^2 + 0x - 1}}$$

Graph your oblique asymptote in the computer viewing window and then carefully sketch

$$f(x) = \frac{2x^2 - 1}{x + 1} .$$



Rational Functions
© Dr. Laura J. Pyzdrowski

CP 2*****

Select any ONE of the 5 functions listed under **Pick one!** and enter an equation for only your choice.

1. $f(x) =$ _____

2. $f(x) =$ _____

3. $f(x) =$ _____

4. $f(x) =$ _____

5. $f(x) =$ _____

If you answer more than one, only the first one answered will be graded.