Logarithms

Names:____

About this Laboratory

An exponential function of the form $f(x) = a^x$, where a is a positive real number not equal to 1, is an example of a one-to-one function. This means that there should exist an inverse function for the exponential function. Such inverse functions are called logarithmic functions. Remember that the exponential function and its inverse function will be symmetric to each other with respect to the line y = x. We will examine the domain, range, and graphs of logarithmic functions. In this laboratory, all answers in decimal form should be to the nearest tenth.

Special Notation for this *Grapher* - log(x) is used for the natural log of x, and log10(x) for the log base 10. Also, we currently have an error in the table. When the logarithm is undefined, the table reports a value of 0.

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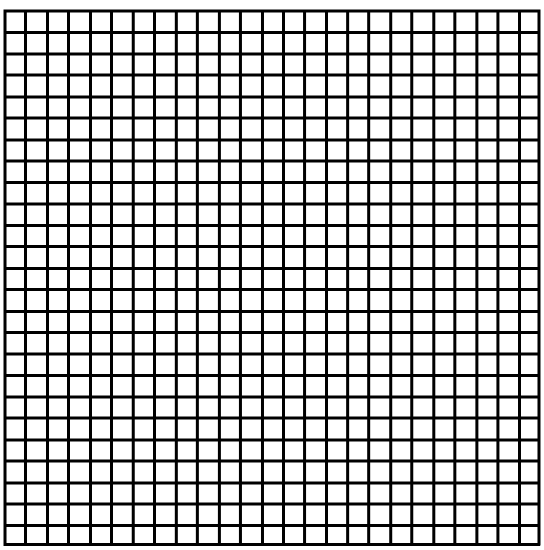
We will examine the function $f(x) = \log_a x$ with base *e*. This is normally written as f(x) = ln(x). However, for this *Grapher*, log(x) is used for the natural log of x.

Sketch f(x) = ln(x).



What is the x-intercept? (a point)

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What is the vertical asymptote? (a line)

____

What are the domain and range?

____

(Be careful when using the computer to evaluate the domain and range of the function.)

For f(x) = ln(x), when x = *e*, y = ___________. When x = 1, y = __________.
(Plug the x values into the equation by hand.)

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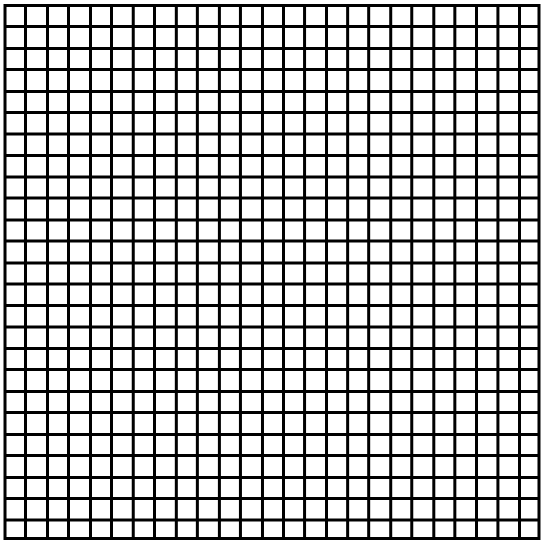
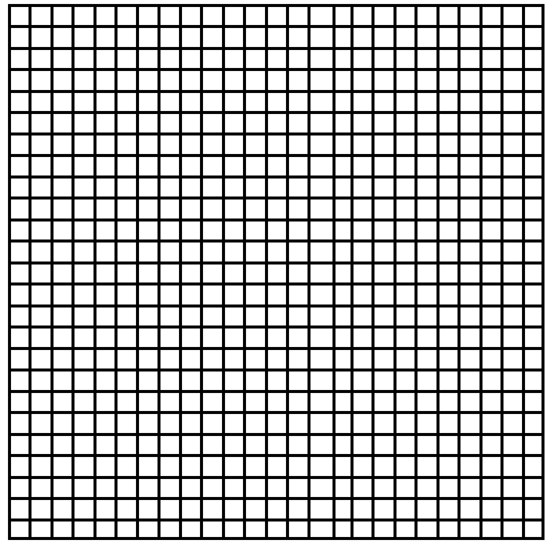
Sketch f(x) = ln(x), g(x) = -ln(x), h(x) = ln(-x), and k(x) = ln(-x+1) on **separate** axes labeling each graph. Give the domain and range of each function. Clearly indicate intercepts and asymptotes on each sketch.



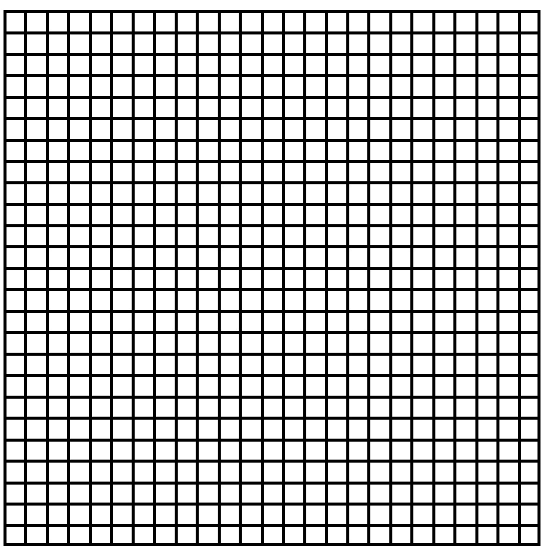


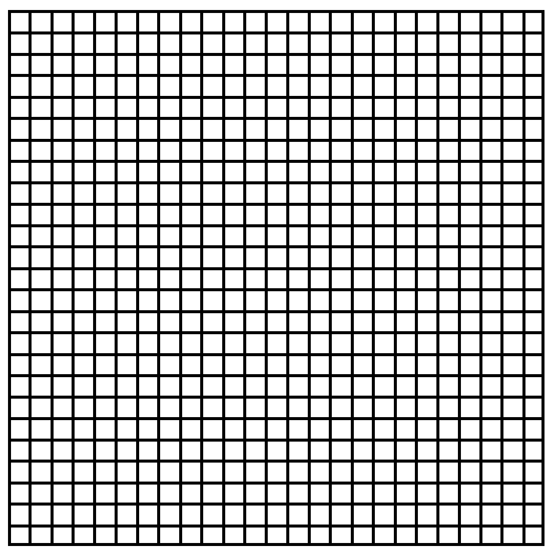
Domain: ________________________

Range: _________________________

Domain: ________________________

Range: _________________________





Domain: ________________________

Range: _________________________

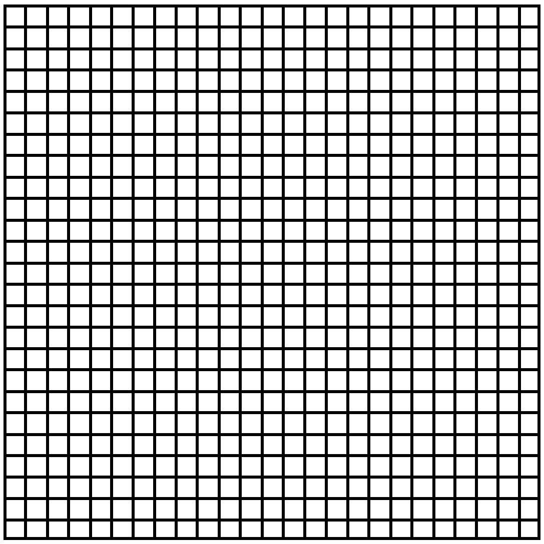
Domain: ________________________

Range: _________________________

CP 2

Let f(x) = e^x. So enter y = e^x onto your screen. Its inverse function will be g(x) = ln(x). Select **Graph function** and then **Graph inverse relation**.

Sketch these two functions and y = x on the same set of axes. Label each function. Clearly give the **domain and range** of f(x) and g(x). Clearly indicate intercepts and asymptotes on the sketch.



Domain_f:____________ Range_f: _____

Domain_g:____________ Range_g: _____

Any horizotal line passes through each function at most one time; therefore, both $f(x) = e^x$ and g(x) = ln(x) are ________________________ functions.

The functions f(x) and g(x) are symmetric to each other with respect to_________________.

Fill in the table. Show analytical work to confirm your entries:
(Plug the x values into the equation(s).)

x	f(x)	x	g(x)
0		1	
1		e	

For $f(x) = e^x$ and g(x) = ln(x). Analytically determine the following:
(Refer to your notes or your book if necessary.)

$g(f(x)) = \ln(e^x) =$ _________________ $f(g(x)) = e^{\ln(x)} =$ __________________

Therefore, f(x) and g(x) are _______________________________________.

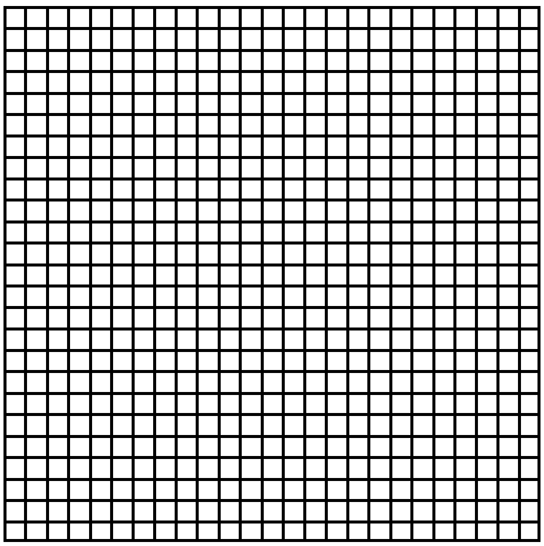
CP 3
We will explore two methods of graphically approximating solutions to equations:

Solve the following logarithmic equation graphically: ln(x) = 2.

Method 1

1. Let f(x) = ln(x) by entering log(x) into the rectangle. Select **Graph**.
   Let g(x) = 2 by entering 2 into the rectangle. Select **Graph**.

A) **Sketch** f(x) = ln(x) and g(x) = 2 on the same set of axes. (Make sure that you can see the intercepts and the point of intersection.) By finding the point of intersection of f(x) and g(x), you are finding the solution to ln(x) = 2.



B) Zoom in on the point of intersection of the two functions to find the x-coordinate of that point to within **.1**.

1. Select the radio button next to **Drag box Zoom In**.
2. Move the arrow inside the graph window and drag a zoom box to include the point of intersection for the two graphs.
3. Continue to zoom in until you can approximate the x-coordinate of the intersection point to the nearest tenth.
4. Select the radio button next to **Coordinates.**
5. Move the arrow inside the graph window and select the point of intersection for the two graphs.

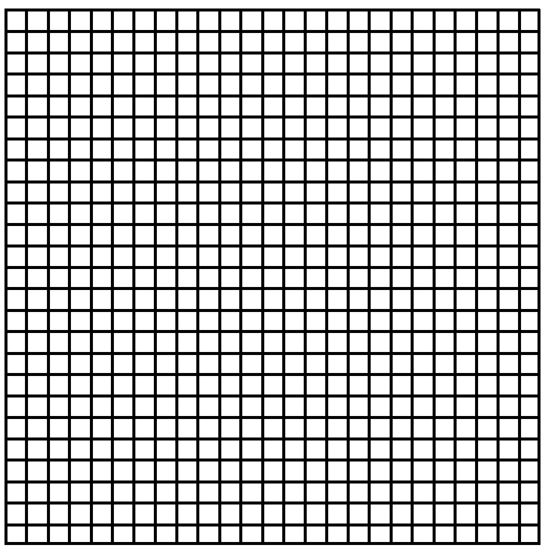
For which **value of x** do the two graphs intersect? ______________

Method 2
Select the **open circle** on the navigation tool of the *Grapher* to reset your scales.

A) Notice that if ln(x) = 2, then by subtracting 2 from both sides of the equation, ln(x)-2 = 0. The second equation is an equivalent equation to the first; and hence, it will yield the same solution for x as the original equation.

Let f(x) = ln(x)-2 by entering ln(x)-2 into the rectangle. Select **Graph**.
Let g(x) = 0 by entering 0 into the rectangle. Select **Graph**.

Sketch f(x) = ln(x)-2 and g(x) = 0.



B) Zoom in on the point of intersection of the function and the line y = 0 to find the x-coordinate of that point to within **.1**. By finding the x-intercept, you are finding the solution to ln(x) = 2.

1. Select the radio button next to **Drag box Zoom In**.
2. Move the arrow inside the graph window and drag a zoom box to include the point of intersection for the two graphs.
3. Continue to zoom in until you can approximate the x-coordinate of the intersection point to the nearest tenth.
4. Use the keyboard arrow keys to trace the point of intersection for the two graphs.

Notice that you are finding the x-intercept of the graph of f(x). This occurs at (_________, 0).

Now analytically solve ln(x) = 2 by changing to an equivalent exponential and then approximating x by using a calculator.

Show work here:

Same computer page, new problem: Now approximate the solution to ln(x) = 1.5 **graphically**. You may use either of the two methods used above. State the method you used and give a solution; include a sketch.

Now analytically solve ln(x) = 1.5 by changing the equation to an equivalent exponential and then approximating x by using a calculator.

Show work here:

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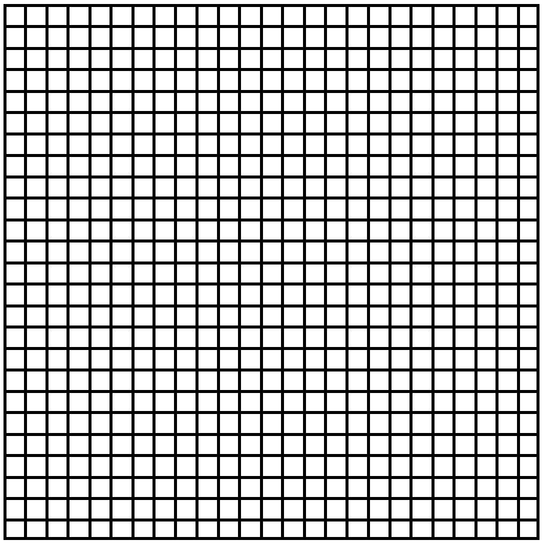
Now let $f(x) = 10^x$, $g(x) = \log_{10}x$ and $h(x) = x$.
(In this lab g(x) = log10(x) but usually you just use g(x) = log(x).)
You may use the *Grapher* to graph one function at a time.

Sketch and label all three functions on one set of axes.



What conjecture can you make about $f(x) = 10^x$ and $g(x) = \log_{10}x$?

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Let's use the *Grapher* to determine how you can graph $y = \log_2 x$ using a graphing calculator. Notice that calculators have the *ln* and *log* keys available.

Recall that the change of base formula is: $\log_b x = \dfrac{\log_a x}{\log_a b}$. So when $y = \log_2 x$, we can change to a base of *e* by letting a = *e* and b = 2. Then $\log_2 x = \dfrac{\ln x}{\ln 2}$. In your *Grapher* window let $f(x)=\dfrac{\log(x)}{\log(2)}$ and let $g(x) = 2^x$. *Remember that in our Grapher log(x) is the syntax used for ln(x).*

Sketch $y = 2^x$, $y = \log_2 x$, and $y = x$ on the same set of axes. Label each function.