

## ACCLAIM MATHEMATICS “Final” Exam

Name: \_\_\_\_\_

(I will send you your test and course grade if you provide me a mailing address.)

July 28, 2006

This examination is open book, open notes. You may use a calculator. If you use your computer as a calculator/word processor, no Google allowed.

1. Give the least positive integer  $x$  that is a solution to the system of congruences

$$x \equiv 1 \pmod{2}$$

$$2x \equiv 1 \pmod{3}$$

$$3x \equiv 4 \pmod{5}$$

2. Let 10,  $x$ , and  $y$  be the three sides of a triangle.

a) Graph the possible pairs of values  $(x, y)$  in the coordinate plane.

b) Highlight the subset of the graph corresponding to isosceles triangles.

3. Give a counterexample if false, or a proof if true: If  $ab \equiv 0 \pmod{15}$  then  $a \equiv 0 \pmod{15}$  or  $b \equiv 0 \pmod{15}$ .

4. Give a counterexample if false, or a proof if true: If  $ab \equiv 0 \pmod{15}$  then  $a \equiv 0 \pmod{5}$  or  $b \equiv 0 \pmod{5}$ .

5. Solve the equation  $3x + 1 = 7$  over  $\mathbb{Q}$  using field properties of  $\mathbb{Q}$ , and solve the same equation over  $\mathbb{Z}$  using appropriate properties of  $\mathbb{Z}$ . Make explicit the algebraic properties you use.

6. Find a polynomial with integer coefficients which has the complex root  $1 + i/2$ .

7. Let  $n$  be a positive integer that is not a perfect square. Let  $\mathbb{Q}(\sqrt{n})$  be the set of all real numbers  $x$  that can be expressed in the form  $x = r + s\sqrt{n}$  where  $r$  and  $s$  are rational numbers. Prove that  $\sqrt{5}$  is not in  $\mathbb{Q}\sqrt{2}$ .

8. One of the students in this class calculated the  $9T + 3 \pmod{26}$  affine transformation ciphertext of his or her name as “ENMMXN TDFZQNA.” Decide whose name this is.
9. See the attached printout from ERIC describing a study conducted about first graders in rural western Idaho. Suppose you want to adapt the study to students in your school district. Identify a mathematical concept that you would expect to be least understood, and explain how you would use topics covered in ACCLAIM mathematics courses to design an in-service activity for the experimental teachers to illuminate this concept.
10. Let  $a$  and  $b$  be positive real numbers. Show that the  $x$  intercept of the line segment joining the points  $(a, a)$  and  $(b, -b)$  is the harmonic mean of  $a$  and  $b$ .