

Math 545 project in lieu of class on September 11

A problem analysis leading to a Diophantine question.

Basic algebra problem: Find two numbers m and n whose sum is 10 and whose product is 18.

- 1) Solve this problem “by hand.”
- 2) Replace 10 by k and solve again, eliminating m and solving for n as a function of k .

Now go to my website and consider the “Diophantine” applet there, corresponding to a more general situation than 2).

Website: <http://math.wvu.edu/~mays/> Click on Applet demos, then Diophantine Problem analysis, nine links up from the bottom of the page.

- 3) Play with the parameter k (which is in the $m+n$ textbox) and observe how the positions of the graphs change as k varies. Figure out how the parabola in the right hand graph depends on the intersection points (or lack thereof) on the left. (Hint: I call it the Viète graph. Viète’s Theorem says that if m and n are solutions to $x^2+bx+c=0$, then $m+n=-b$ and $mn=c$.) You can enter the parameters directly in the text boxes, or you can skip by 1’s instead of by 0.1’s if you hold down the shift key while you are clicking on the increment/decrement buttons.
- 4) Find bounds $s < k < t$ so that the line with equation $x+y = k$ does not intersect the hyperbola $xy = 18$. Numerically approximate the bounding values of k , and observe what happens to the graphs as those values are approached.
- 5) Explore the effect of varying the other parameter, corresponding to the product of m and n .

Now let us consider the corresponding Diophantine equation. A Diophantine equation is restricted to having integer solutions. Change the combo box at the top to request integer solutions instead of real numbers. Now the graphs on the left have lattice points (points with Cartesian coordinates that are integers) highlighted, and the graph on the right lights up the intercepts if they happen to be integers.

- 6) Skip through integer values of $m n$. Predict how many points on the red graph are highlighted based on the arithmetic nature of the integer $m n$.
- 7) The blue graph on the left appears to be “all or nothing.” Figure out what I mean by this, and argue why it should be so, based on its slope.

- 8) What happens on the right when the point(s) of intersection of the line and the hyperbola on the left are lattice points?
- 9) Note the symmetry of the location of lattice points on the left hand graph, and account for it.
- 10) List all choices for m and n with $0 \leq m, n \leq 25$, for which solutions to the Diophantine problem exist.
- 11) Discuss the configuration of the graphs on the left, and the graph on the right in the case when $m = n$, especially in the Diophantine case.