Review Sheet: Chapter 4

Content: "Essential Calculus, Early Transcendentals," James Stewart, 2007
Chapter 4: Applications of Differentiation

Concepts, Definitions, Laws, Theorems, Formulae:

(def) A function f has an **absolute maximum** at c if $f(c) \geq f(x)$ for all x in the domain of f. The number $f(c)$ is called a **maximum value** of f.

(def) A function f has an **absolute minimum** at c if $f(c) \leq f(x)$ for all x in the domain of f. The number $f(c)$ is called a **minimum value** of f.

(def) Maximum and minimum values of a function f are generally called **extreme values** of f.

(def) A function f has a **local maximum** (aka relative maximum) at c if $f(c) \geq f(x)$ when x is near c. The word "near" implies that you can approach c from both left and right directions.

(def) A function f has a **local minimum** (aka relative minimum) at c if $f(c) \leq f(x)$ when x is near c. The word "near" implies that you can approach c from both left and right directions.

(con) Relative max and mins cannot occur at endpoints. However, absolute max and mins can occur at endpoints.

(thm) **The Extreme Value Theorem**: If f is continuous on a closed interval $[a,b]$, then f attains both an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some values c and d that are inside the interval $[a,b]$.

(thm) **Fermat's Theorem**: If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

(def) A **critical number** of a function f is a number c in the domain of f so that either $f'(c) = 0$ or $f'(c)$ does not exist.

(con) If f has a local maximum or local minimum at c, then c is necessarily a critical number of f.

(thm) **Rolle's Theorem**: Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a,b]$.
2. f is differentiable on the open interval (a,b).
3. $f(a) = f(b)$

THEN, there is a number c in (a,b) so that $f'(c) = 0$.
(Definitely, be able to draw a picture of what this theorem is saying.)

(con) Rolle's Theorem is a specific case of the Mean Value Theorem.

(thm) **The Mean Value Theorem**: Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a,b]$.
2. f is differentiable on the open interval (a,b).

THEN, there is a number c in (a,b) so that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

(Definitely, be able to draw a picture of what this theorem is saying.)

(thm) If $f'(x) = 0$ for every x in an interval (a,b), then f is a constant value along (a,b).

(con) If $f'(x) = g'(x)$ for all x in an interval (a,b), then $f(x)$ and $g(x)$ remain a constant distance apart throughout (a,b).

(con) Mean Value Theorem tells us that if you calculate the slope of a secant line between two points along a 'nice' function f, that at some point between the two, you must be able to create a tangent line along f that has the same slope. (And, it might happen more than once!)

(con) **Increasing/Decreasing Test**:

1. If $f'(x) > 0$ on an interval, then f is increasing on that interval.
2. If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

(con) **The First Derivative Test**: (this is a big one)
Suppose that c is a critical number of a continuous function f.

1. If f' changes from positive to negative at c, then f has a local maximum at c.
2. If f' changes from negative to positive at c, then f has a local minimum at c.
3. If f' does not change sign on either side of c, you don't know what is happening at c.

(con) **Concavity Test**:

1. If $f''(x) > 0$ on an interval, then f is concave up (like a cup) on that interval.
2. If $f''(x) < 0$ on an interval, then f is concave down (like a frown) on that interval.

(def) A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave up to concave down, or concave down to concave up.

(con) **The Second Derivative Test**: (another big one)
Suppose that c is a critical number of a continuous function f.

1. If $f''(c) > 0$, then f has a local minimum at c. (c is at the bottom of the cup)
2. If $f''(c) < 0$, then f has a local maximum at c. (c is at the top of the frown)

(form) **Newton's Method**: To approximate the root of a function, you can iteratively solve using the equation:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Be able to show how you can derive this equation. A picture is ok.

(def) A function F is called an **anti-derivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

(thm) If F is an anti-derivative of f on an interval I, then the most general anti-derivative of f on I is $F(x) + C$ where C is an arbitrary constant.

(con) The anti-derivatives of functions can be found by thinking through the derivative rules in reverse.

(process) How to find absolute max or min values of a continuous function f on a closed interval $[a,b]$:

1. Evaluate the derivative of f.
2. Find the critical numbers of f, making sure that they live within the interval (a,b).
3. Evaluate $f(a)$, $f(b)$, and $f(\text{valid critical numbers})$.
4. Label the largest value of f as the absolute max and the smallest value as the absolute min.

(process) How to accurately sketch a curve without a calculator:

1. Evaluate domain.
2. Find x and y intercepts.
3. Evaluate symmetry.
4. Find vertical and horizontal asymptotes.
5. Evaluate first derivative and find critical points.
6. Evaluate second derivative and find possible inflection points.
7. Set up a sign chart/table and track the components of the first and second derivatives, so that you can assess intervals of increase/decrease/concave-up/concave-down.
8. Determine particularly where your local min/max are and inflection points.
9. Draw a set of axes right below your sign chart and line up with your x-values.
10. Mark explicitly all of the points that you know, and sketch in HA and VA.
11. Use your sign chart and draw in the generalized shapes that the definitions demand, connecting the points that you know with the nature of asymptotes and symmetry.

(process) How to approach optimization problems:

1. Draw a decent picture of the problem and identify any equations that occur naturally from the picture or the problem's textual description.
2. If there is more than one equation, determine which one is to be optimized, then use subsidiary equations to write the one to be optimized in terms of a single variable.
3. Evaluate the first derivative of the equation you want to optimize and find critical points.
4. STOP and THINK. Do these critical points make physical sense? Chuck the ones that do not.
5. Find the second derivative of the equation you want to optimize.
6. Evaluate the second derivative at the critical points (and any end points that might make physical sense) to determine max or min.
7. Go back and make sure that you have actually answered any question given in the problem statement.

NOTE TO THE WISE: Think about the algebra before you go willy nilly taking derivatives of functions. Always think to yourself, "is there a way to do this that is easier?" and avoid adding difficulty to a simple problem.