SAMPLE TEST # 1

Solve the following exercises. Show your work. (No credit will be given for an answer with no supporting work shown.)

Ex. 1. Each of the following differential equations is of one of the following form: linear, separable, homogenous, Bernouli, or exact. Solve each of these using appropriate method. (In the actual test, in some of such problems you will be asked only to label the differential equation as one of these five kinds, including justification for the label, without providing full solution.)

(a) \( \frac{dy}{dx} = \frac{e^x - e^{-x}}{3 + 4y}, \ y(0) = 1 \)

(b) \( \frac{dy}{dx} + 6x + (\ln x - 2) \frac{dy}{dx} = 0, \ x > 0 \)

(c) \( ty' - y = t^2 e^{-t}, \ t > 0 \)

(d) \( \frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy} \)

(e) \( x^2y' = y^3 - 2xy, \ x > 0 \)

(f) \( \frac{dy}{dx} + y = \frac{1}{1 + e^x} \)

Ex. 2. Draw the direction field for the equation \( \frac{dy}{dx} = 1 + y^2 \). In your drawing show the places where the slopes are 1, 2, and 5.

Ex. 3. Without solving, determine the largest interval in which the initial value problem \((x^2 - x - 6)y' + y \cos x = e^x, \ y(2) = 0\), has a unique solution.

Ex. 4. Apply Euler’s method to \( y' = \frac{4-ty}{1+y^2}, \ y(0) = -2 \), with step \( h = 0.1 \) to estimate \( y(0.1) \).

Ex. 5. A tank with a capacity of 500 gal originally contains 200 gal of wather with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flaw out of the tank at the rate of 2 gal/min. Write down an initial value problem (ODE plus initial condition) giving the amount of salt in the tank at any time during the first hour. Do not solve the equation. Remember to give the initial condition.