MATH 251
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# SAMPLE TEST # 4

Solve the following exercises. **Show your work.**

**Ex. 1.** Let $\mathbf{F}(x,y,z) = \ln(xy)\,\mathbf{i} + (x+\sin z)\,\mathbf{j} + (2y-2z)\,\mathbf{k}$. Calculate $\operatorname{div}\mathbf{F}$ and $\operatorname{curl}\mathbf{F}$.

**Ex. 2.** Evaluate $\int_C xy\,ds$, where $C$ is the parametric curve for which $x = 3t$, $y = t^4$, and $0 \le t \le 1$.

**Ex. 3.** Evaluate the integral, where $C$ is the graph of $y = x^3$ from $(-1,-1)$ to $(1,1)$.

$\int_C y^2\,dx + xdy =$

**Ex. 4.** Determine if the following vector field is conservative. Find potential function for a field, if it is conservative.

(a) $\mathbf{F} = \left(x^3 + \frac{y}{x}\right)\mathbf{i} + (y^2 + \ln x)\mathbf{j}$

(b) $\mathbf{F} = (y\cos x + \ln y)\,\mathbf{i} + \left(\frac{x}{y} + e^y\right)\mathbf{j}$

**Ex. 5.** Evaluate the integral

$$\int_{(\pi/2,\pi/2)}^{(\pi,\pi)} (\sin y + y\cos x)\,dx + (\sin x + x\cos y)\,dy =$$

**Ex. 6.** **Apply Green's theorem** to evaluate the following integral, where the simple closed curve $C$ is the boundary of the circle $x^2 + y^2 = 1$.

$\oint_C (\sin x - x^2 y)\,dx + xy^2\,dy =$

**Ex. 7.** Find the area of the surface of the graph of $z = x^2 + y$ that lies above the triangle in the $xy$-plane with vertices at $(0,0)$, $(1,0)$, and $(1,1)$.

**Ex. 8.** Evaluate

(a) $$4\begin{bmatrix} 1 & -1 & 5 \\ 1 & 3 & 2 \\ 0 & 3 & -2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 & -2 \\ 3 & 1 & -1 \\ -1 & 0 & 11 \end{bmatrix} =$$

(b) $$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 7 & 0 \\ 1 & 2 \end{bmatrix} =$$

**Ex. 9.** Find the inverse matrix of

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$